LEARNING MATHEMATICS BY SYMMETRY

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There is a well known symbiotic relationship between the notion of symmetry and the domain of mathematics: Mathematics provides tools for the description and use of symmetry; and symmetry is an efficient heuristic for the solution of problems in virtually any domain of mathematics. Efficiency, however, is but one aspect of the relationship between symmetry and mathematics; a possibly stronger reason for mathematicians to describe and use symmetry is its aesthetic appeal. For example, our research has indicated that problems involving symmetry are inherently appealing to mathematicians.

The existence of such an intimate relationship between symmetry and mathematics suggests that symmetry also has a role to play in the processes of learning and teaching mathematics. It provides a tool to structure mathematical content (also, and especially, outside of geometry), so as to make it more accessible to the student and easier to explain for the teacher. For example, inverse functions and their derivatives are often taught (and learned) in a formal algebraic manner which does not allow students to fill the notions with intuitive content. But these notions rely heavily on the symmetry operation of inversion, and on the fact that this operation conserves some global properties of the function. Using a graphical approach and linking it closely with the algebraic one, these global properties can be emphasized and, in fact, made the basis and the heart for a deep understanding of the entire topic.

At the conference, we will present additional examples and discuss their common features; these point to possible cognitive reasons why symmetry favours learning, such as proximity to intuitive preconceptions and a reduction of the cognitive load. But we will close by making the point that the main rationale for the use of symmetry in teaching and learning mathematics, just as the main reason for the use symmetry in mathematics itself, is aesthetic.