THE META-MORPHOLOGY OF POLYHEDRAL CLUSTERS

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This paper continues our meta-morphological modelling of orderly space structures and extends our thesis *Structures on Hyper-Structures* [1] to classes of inter-transforming polyhedral clusters.

As an example, a family of 256 inter-transforming clusters of icosahedral symmetry and composed of icosahedral polyhedra can be generated from the family of 16 inter-transforming polyhedra of icosahedral symmetry described in [2]. The concept was briefly described in [3-5] where a family of 64 icosahedral clusters was suggested. The 16 polyhedra are used as *vertex-polyhedra* and are placed on the vertices of an underlying *network-polyhedron* which is also selected from any one of the 16 polyhedra. The 256 clusters are obtained from all combinations of the vertex-polyhedra and network-polyhedra. The clusters, like the polyhedra themselves, are binary-coded and can be arranged on the vertices of a 8-dimensional cube, in the manner of our previous work. Continuous transformations of the clusters to one another are possible within this 8-cubic meta-structural space. A portion of a subset of 64 clusters, arranged in 6-cubic space, is shown in Figure 1.

The process of replacing each vertex with a vertex-polyhedron can be continued recursively, leading to a fractal polyhedron which can be characterized by a fundamental region within a fundamental region within a fundamental region [5]. In mirror-symmetric space structures, the recursive fundamental region is a fractal kaleidoscope. The vertex-polyhedra and the net-polyhedra have the same orientation, thus displaying another level of self-similarity in the packing.

We note that by changing the symmetry of the vertex-polyhedra and the net-polyhedra to the tetrahedral and octahedral families, a much larger family of polyhedral clusters is obtained. We also note that while only symmetric clusters are shown, the clusters can be extended by extending the underlying net to fill space. Interesting cases are non-periodic packings of icoshaedral polyhedra where the vertex-polyhedra are placed on the vertices of space-filling rhombohedral cells, e.g. the two well-known golden rhombohedra and others as described in [6].

References :

1. Lalvani, H., *Multi-dimensional Periodic Arrangements of Transforming Space Structures*, Ph.D. Thesis, University of Pennsylvania, (1981); publ. by University Microfilms, Ann Arbor (1982); See also : *Structures on Hyper-Structures*, Lalvani, New York (1982).
2. Ref [1], p.109.
3. Ref [1], p.206-7 and p.219 (Fig.157).
4. Lalvani, H., *Patterns in Hyper-Spaces*, Lalvani, New York (1982); p.24.
5. Lalvani, H., Non-periodic Space Structures, *Space Structures* 2 (1986-87),p.105.
6. Lalvani, H., Non-periodic Packings of Icosahedral Polyhedra, *Space Structures* 2 (1986-87) 177-180.

Figure: A portion of 64 icosahedral clusters arranged in 6-cubic meta-structural space.
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