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Groups of Symmetry as the Mathematical Representations of Global Physical Laws in Cosmological Models of the Universe

Abstract

0.

As it is well known since antiquity, many of objects from around us reveal the specific feature called symmetry. Ancient philosophers of nature recognized, that in case of some objects, there exist a possibility of turning or moving them in such the way, that after the process, objects seem to be exactly the same as before. For example, the sphere stays insensible for any possible rotations over the axes which cross its geometrical center.

Classical physicists of next generations knew about symmetries in nature much more. They had specified and defined different types of symmetrical features of objects. The chemists and 'solid state' physicists had applied this knowledge to the atomic structures, what allowed really unexpected increase of technologies in the end of XIX-th and at the beginning of XX-th century.

But in the course of years, after relativistic and quantum revolution in science, the idea of symmetry took the new importance, which changed our understanding of the universe as a whole as well as understanding of the same term: "symmetry". Recognition of physical reality today, to a large degree is equivalent to examining the features of elements of so called 'symmetry groups', which gather symmetrical features of this reality and may be understood as specific mathematical representations of important physical laws. By the same way, there changed the understanding of a role of same mathematics in the process of recognition. Mathematics today is not only 'a tool' of physics or other branches, but comes to be the internal logical structure of science (some say it is the only reality of science). This paper is intended to bring closer the issue.

1.

The recognition of physical reality is possible thanks to two highly surprising and unreasonable properties of the universe. The first one, sometimes called ‘mathematization’, allows us to find mathematical objects (functions) which are connected with separate physical facts. Such the connections are of the one-to-one function type, i.e. strongly analytical one. The second property of the universe is so called ‘idealization’. This one may be understood as a possibility of emulating the physical reality or its part by an appropriate defined set of functions, number of which may be finite and usually much less than the number of links and relationships between separate elements of same reality. Thanks to it, there is possible to construct mathematical models of the world, which models are much less complicated than physical reality and fully analytical as well. On the other hand, models behave like the world they are modeling and may be examined with use of mathematical tools. In other words, the two properties of the universe, 'mathematization' and 'idealization', make possible the recognition of the universe. The hypothetical world without these features would be, as Einstein called, the ‘malicious’ one. Even worlds in which the feature of ‘idealization’ exists and all the functions representing physical facts are smooth enough and only a little too complicated in arithmetic sense (i.e. the physical reality is only ‘partly malicious’), wouldn’t be recognizable without fundamental problems. Our universe, fortunately, seems to be fully 'no-malicious', thanks to what constructing, examining and verifying the models is possible and relatively easy.

The process of constructing the model usually starts with a trial of finding these mathematical objects which are able to represent correctly some observed physical facts from examined area of reality (Fig.1). Later, after verifying the result, one must find the mathematical environment (space) appropriate for objects found before. If this succeeded, the next step is defining boundary conditions for the structure of a new one model, for specifying the edges of it. Now the model may be examined in an analytical or methodological way for finding its important features, which, thanks to the property of 'mathematization', do represent following facts in physical world. Of course, these facts must be finally verified in an empirical way.

Such the recognition of reality with use of mathematical models is the only way of recognition available in physics and natural sciences. In fact, the only examined object in this process is the specific mathematical structure (the model) which represents appropriate part of the universe. The structure behaves almost exactly like the physical reality, being at the same time analytical, easy as well as subtle like it. This is why, recognition of 'malicious' worlds wouldn't be possible. If the functions which represent physical facts comes to be not smooth (analytical) enough, the model starts to emulate many realities at the same time or doesn't emulate any physical world at all. Except of this, there doesn't exist the possibility of unmistakable verifying the results, even if there are any (see Fig.2).

As one may expect, each of the model's elements, both of the formal (for example: the value of locally defined tensor), as well as methodological (assumption, that the model's space shall be linear one) kind, has got its strict physical importance. Physicists often say: "let's take four-dimensional Riemannian manifold" when they want to think about the real physical space-time. In fact, thanks to the property of 'mathematization', they are not wrong in saying that. Hence one may expect as well, that global substructures of the model, potentially existing inside its inside, shall imply existence of some global physical features of the universe.

2.

Understanding the world consists not only in discovering new physical facts, but, first of all, in recognizing the structure of relationship among them. To indicate this structure, one must define the model's mathematical substructure, which potentially may be representation of it.

One of the common and most important mathematical objects which may be useful in this case, is algebraic group. The couple (A, f), where A is a set of elements and f is a feature (sometimes called 'action'), comes to be a group when one's satisfied what follows: (i) (afb)fc = af(bfc), where a,b,c are elements from the set A, (ii) there exist an element e, which satisfies afe = efa = fa, (iii) for every element a there exist the element a', which satisfies afa' = a'fa = e. For example, real numbers and the operation of addition, constitute the group. It is easy to notice, that some geometrical transformations may create other groups (for example, the group may be constituted by the set of isometries of the surface and the operation of addition).

At the beginning of subsection 0, it's been mentioned the symmetry understood as the feature of physical objects. One's easy to recognize, that transformations which lead the object to itself may create the group with respect to their addition. Such the group may be very ordinary, when having only one element, or more complicated, when having more of them. There are possible two different kinds of such the groups. The first one is connected with global (the same everywhere) transformation parameter. Mentioned rotations of the whole sphere over the axes which cross its geometrical center may be an example of such the transformations. The second kind of the group is connected with parameter of transformation which is been changing with move through the layout. Here the example may be the revolution of a cylinder. The figure doesn't change its shape even if the angle of which the cylinder is being rotated changes along the main axis of symmetry (the axis crossing geometrical center of cylinder and perpendicular to its basis), see Fig. 3.

Possibility of defining such the groups of symmetry is very useful for all the branches of technology, physical chemistry and solid state physics. But their importance increases rapidly in cosmology and quantum mechanics.

3.

Exactly as defining the group for the set of symmetrical transformations of physical objects is possible, as well possible is the same operation regarding to elements of the model. These elements are mathematical objects, but this doesn't change the situation, because there exists a set of appropriate 'symmetrical' transformations of them (mathematical objects are invariants of specified transformations), and, thanks to the mentioned nature's property of 'mathematization', it is possible to treat these mathematical objects in an equivalent way to physical ones.

Such defined groups of symmetry may be of global or locally marked kind. Most important for understanding the whole universe are the groups of global kind in advanced cosmological models. As it's been told before, one may expect, that such the groups will be representation of some important global physical features of the nature. In fact that is true. The theorem (proved by Noether) on relationship between the symmetry and principles of conservation allows us to follow this idea. Finally we are convinced, that existence of groups of symmetry at the model's structure implies existence of appropriate physical features (physical principles) of reality which is being modeled.

In case of modern advanced global model of universe (cosmological one), there are being indicated three important groups of symmetry, appropriately of 1, 3 and 8 elements (generators). They are responsible for global laws of conservation of charges and momentum, laws well known yet in classical (Newtonian) physics, but discovered in an empirical way. The same groups, when regarding the local invariance, imply physical relationships for boson and gluon fields. It's possible, because generators of the groups represent appropriate physical features of fundamental particles. The number of generators is strict connected with the number of features. The last properties of space-time wouldn't be, of course, discovered in any of empirical methods. In other words, there wouldn't be possible understanding the essence of nuclear processes without suitable use of theory of symmetry groups.

4.

From examples above, one may lead main conclusions:

- thanks to the property of 'mathematization', one may use mathematical objects called groups of symmetry for discovering and indicating separate global features of the universe,
- this way of recognizing the universe is the only possible (at least - up to now),
- the term 'symmetry' means much more than only a physical feature of being invariant for any of transformations; it means exactly the same, but in a meta-objective sense,
- the world we live in is strongly symmetrical at its basis; all the physical principles of the universe are in origin of symmetrical type,
- Einsteinian understanding of the world as 'non-malicious' one is a manifestation of what above.

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