

USING ASYMMETRICAL PROBABILITY DENSITY FUNCTIONS (PDFS) IN SIMULATIONS

R.R. (Ron) Goforth, Ph.D.
Computer Systems Engineering
University of Arkansas
Fayetteville, AR 72701 USA
e-mail: rrg@engr.uark.edu

Simulations, even where valid models exist, are frequently limited by the availability of good data. This is particularly true in an area of current global concern, namely, models for forecasting the future incidence of HIV/AIDS.

Epidemiological models and their corresponding computer simulations can be used to determine the effects of varying assumptions concerning the mechanisms for the spread and containment of the AIDS pandemic. Populations susceptible to AIDS, defined as sub-populations of the general population, may be considered to be distinct classes within which the number of AIDS cases propagate. The use of transmission categories allows definition of distinct susceptible populations for which independent growth curves can be calculated. Since crossover among these populations occurs, it is also necessary to provide for dynamic interactions among them in the calculation of total incidence rates.

Data which are only partially quantified play a major complicating role in the significant problem of forecasting the future course of the AIDS pandemic. For example, the likelihood of HIV infection is a function of both the risk of infection per exposure and the frequency of exposure. Factors such as these can best be described in terms of probabilities, and these probabilities typically are difficult to estimate.

Current modeling and computer simulation tools do not effectively incorporate partial information. Modelers are therefore forced to choose arbitrary numbers in order to use existing modeling and simulation technology. In these cases, the reliability of the scenarios generated becomes questionable. Never-the-less, simulation is a promising tool in predicting the future course of the AIDS pandemic particularly in studies of the relative impacts of alternative public health strategies.

The the focus of the work described here relates to logistic population growth in one mode of HIV transmission, specifically transmission through intravenous drug abuse, in an HIV/AIDS epidemiological model. A typical resulting logisitic forecast is given in Figure 1. Simulations based on this model use "poorly quantified" data derived from historical sources and estimations of an upper bound (maximum size of the susceptible population).

Recent research has provided insight into the implications of using asymmetrical probability density functions (pdfs) as contrasted to uniform or normally (symmetrically) distributed data. This is an important consideration since many of the critical determining factors in HIV/AIDS epidemiology are known to be skewed even if of uncertain mean values.

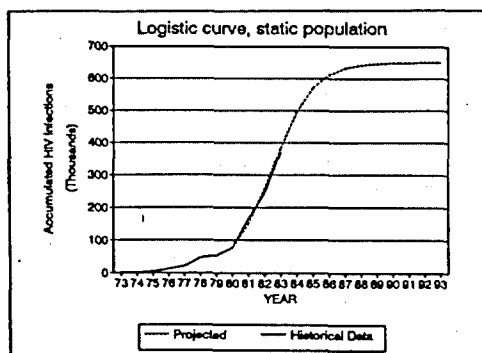


Figure 1. Logistic growth of accumulated infections within a static population of susceptibles.

The products of an exact value and a symmetrical probability density function (pdf) or of two symmetrical pdfs are symmetrical. The iterated products of asymmetrical pdfs with either other asymmetrical or symmetrical pdfs may exhibit unexpected properties. These properties may have profound effects on the behavior of complex models. In the domain under consideration, population and epidemiological modeling, the underlying mechanisms dictate that asymmetrical pdfs be used (e.g. determinants of mortality and natality across age cohorts or the rate at which a population undergoes certain demographic changes).

A model and simulation that accommodates a dynamically changing population of susceptibles, with asymmetrally distributed "refresh" rates (R), yields qualitatively different growth curves as shown in Figure 2. Note the shifting minima and maxima

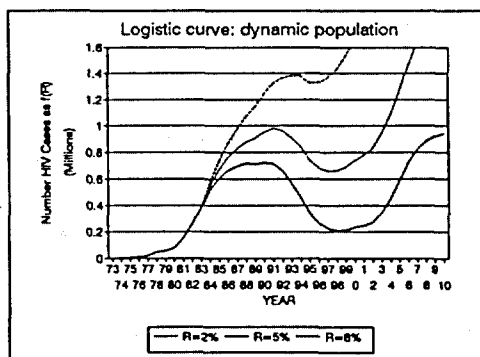


Figure 2. Logistic growth within a range of dynamically changing populations of susceptibles.

Since data about the HIV/AIDS pandemic often is incompletely quantifiable, and cannot be expressed as simple numerical values, the use of other formalisms for expressing partially quantified knowledge is critical. As these modes are added to a simulation, novel risks may be introduced. These risks may be associated with limited understanding of the proper use of semi-quantitative data, the issue of data independence or the presence of correlated variables, and the implications of alternative mathematical operations on variables representing asymmetrally distributed values.