GEOMETRIC SYMMETRY, MELODIC SYMMETRY

John W. Emert
Department of Mathematical Sciences, Ball State University
Muncie, Indiana 47306-0490, U. S. A.
Email: emert@math.bsu.edu

The structures used to describe geometric symmetries in a plane pattern can be adapted to analyze the melodic symmetries of motifs, melodies, and fugues. Just as the locations of a recurring geometric motif within a plane pattern can be associated by combinations of translations and reflections, the occurrences of a musical motif within a composition can be associated by translations and reflections in both pitch and time. The possible symmetries in pitch-time are a proper subset of the plane symmetries. This paper will develop the basic tools for building symmetries in the plane, establish the possible symmetries in pitch-time, and present geometric and melodic examples of such symmetries.

A plane isometry (or rigid motion) is a mapping of the plane onto itself which preserves the relative distance between pairs of points. That is, any pair of points is mapped to points which are the same distance apart. Examples of plane isometries include reflections about a line and rotations about a point. It will be shown that (1) every plane isometry is characterized by the movement of any three points, (2) every plane isometry can be achieved as the composition of at most three reflections about lines, and (3) the composition of at most three reflections results in one of five possible types of plane isometries: no movement, reflection

about a line, translation along a line, reflection about and translation along the same line, and rotation about a point.

While such global symmetries can sometimes be found in fugues and twelve-tone compositions, more often localized symmetries are identifiable. A musical theme can be constructed from a short motif by combining, at various pitches, the motif, its inversion (inverting the relative pitches of the motif) , its retrograde (reversing the order of the pitches), and its retrograde inversion (inverting the pitches and reversing their order). The repetition of the motif in these various ways can be described using the language of geometric isometries. Some compositions allow such an analysis at the higher level of theme or twelve-tone row repetition.

Since the two dimensions of pitch and time are measured in different units, some of the symmetries of the plane (*e.g.,* rotations other than half turns) are difficult to characterize. By considering pitch and time as independent variables, sixteen different pitch-time isometries can be formed. These categories collapse to twelve of the seventeen plane isometries.

The interlacing geometric structures with pitched sound provides a structure on which to weave auditory designs. The presentation includes geometric and melodic examples of such symmetries. Does music actually result from this exercise? I defer this question to the listener.

References

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