

SYMMETRY OF PATTERNS AND PATTERNS OF SYMMETRY

You are aware that the system of cards which Jacquard invented are the means by which we can communicate to a very ordinary loom orders to weave any PATTERN that may be desired. Availing myself of the same beautiful invention, I have by similar means communicated to my calculating engine orders to calculate any formula however complicated; but I have also advanced one stage further, and I have communicated through the same means orders to follow certain laws in the use of those cards, and thus the calculating engine can solve any equations, eliminate between any number of variables, and perform the highest operations of analysis.

Charles Babbage's letter to Arago, the French physicist, December 1839 (quoted after Randell, B., ed., *The Origins of Digital Computers: Selected Papers*, 3rd ed., Berlin: Springer, 1982, p. 59).

A mathematician, like a painter or a poet, is a maker of PATTERNS. If his [or her] PATTERNS are more permanent than theirs, it is because they are made with ideas. A painter makes PATTERNS with shapes and colours, a poet with words. [...] The mathematician's PATTERNS, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.

G. H. Hardy (*A Mathematician's Apology*, Cambridge: University Press, 1967, pp. 84-85; first ed., 1940).

PATTERNS play a significant role not only in arts, but also in science and technology, including the invention of the first programmable computer (calculating engine) that possesses a memory and a possible style of thinking in mathematics. The letter by Charles Babbage, whose mechanical engines anticipated the architecture of the electronic computers more than 100 years earlier, and the quote by a leading number theorist, G. H. Hardy, demonstrate this fact impressively. Apropos, some artists would disagree that the mathematician's pattern are more permanent than theirs. Indeed, the artist's "patterns of ideas" (theory of arts, aesthetics, etc.) may be as permanent as the mathematician's, only the art works, as physical objects, are less permanent: the number of mathematical theorems that survived from ancient times is greater than the number of paintings, sculptures, buildings, or authentic manuscripts of literary works and musical compositions.

The English word "pattern" came from the Old French *patron*, which was originated in the Latin *pater*, father. Thus pattern means a person or thing so ideal as to be worthy of imitation or copying. This understanding also led to such meanings as model, guide, plan, sample, and even tendency or characteristics (cf., behavior patterns).

For us, however, the most important meaning of this word is connected with arrangement of form, disposition of parts or elements, and design or decoration. This meaning is applicable in very many artistic, scientific, and technical fields, moreover the expression "pattern", whose original understanding is connected with ideal things, often associated with symmetric arrangements, such that regular systems of figure, periodic structures, wallpaper designs, etc. We may illustrate the use of this expression with various publications. Thus, P. A. MacMahon's classical book *New Mathematical Pastimes* (Cambridge: Cambridge University Press, 1921) includes a part "Repeating patterns". The geometric monograph by Grünbaum and Shephard is entitled *Tilings and Patterns* (New York: Freeman, 1987). We may cite two anthropological books dealing with symmetry analysis of ornaments: Zaslow and Dittert's work *Pattern Mathematics and Archaeology* (Tempe: Arizona State University, 1977) and Washburn and Crowe's book *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis* (Seattle: University of Washington Press, 1988). The book by P. M. Stevens is written for designers: *Handbook of Regular Patterns: An Introduction to Symmetry in Two Dimensions* (Cambridge, Mass.: MIT Press, 1980). The expression "pattern" is also popular in biomorphology, see, for example, the monograph by Portmann *Animal Forms and Patterns: A Study of the Appearance of Animals* (London: Faber and Faber, 1952). The specific use the same expression can be illustrated by the geologists' tectonic patterns, the radar specialists' cosecant-squared pattern (the antenna gain should be proportional to the cosecant-squared of the elevation angle of the possible targets), etc. Pattern recognition is investigated in both psychology and computer science, specifically in connection with artificial intelligence (also see the related mathematical works, e.g., U. Grenander's monographs *Lectures in Pattern Theory*, vols. 1-3, New York: Springer, 1976-1981). We realized that the "symmetric pair" of our symposium-title also exists: *Patterns of Symmetry*, Northampton, Massachusetts, February 1973 (its volume was edited by Senechal and Fleck, Amherst: University of Massachusetts Press, 1977; Russian translation, *Uzory simmetrii*, Moskva: Mir 1980).

The results in non-linear mathematics (chaos, catastrophes, fractals) and computer graphics lead to a direction in science and art which are strongly connected with the "beauty of patterns". Many of them can be associated with bifurcation and other symmetry breakings. Of course it would be a mistake to simplify the teaching of mathematics and computing just for making nice patterns (there was even a mathematical debate about such questions between "antifractalists" and "fractalists"), but this interdisciplinary field may significantly popularize various mathematical problems, where the students need some inspiration.

In the arts, pattern is a standard expression, especially in design. There are some forums for it, see, for example, the *International Triennial of Patterns* in Budapest. The patterns of literary works or musical compositions, which can be visualized in diagrams, are also investigated.

All of the discussed approaches are connected with more or less geometric patterns. The earlier quotation of the British mathematician G. H. Hardy, however, gave a broader interpretation of the concept: patterns of ideas. In this sense pattern making is a very important aspect of creativity in science, art, and technology. We should add that there is a special Irish meaning of the word "pattern": it is a feast or merrymaking in honor of a patron. Let this symposium on patterns be also a feast in honor of symmetry as a patron of the connections between art and science

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