

CRYSTALLOGRAPHIC INTERPRETATION: RECURRENCE FORMULA OF NATURAL NUMBERS' POWER SERIES SUMMATION

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Some geometrical interpretation of n -th power series of natural numbers' summation were presented for $n = 1, 2, 3$ (Gardner, M 1986). However, its generalized recurrence formula has not been proposed so far as the author is aware. In this paper a new geometrical scheme for the summation is presented by using the geometry of the closest packing of circles (one- and two-dimensional cases) and that of spheres (three-dimensional case) which is called face-centered close packed structure. It is shown that higher-order power series of natural numbers can be obtained by considering further extension of the dimension, namely the lattice points in j -dimensional regular by hyper- simplex α_j . The crystallographic interpretation of the recurrence formula of the summation is presented by the superposition of a_j , in which a_j stacks with a linear increment weight of the lattice points.

The superposition of α_j is carried out every $(j+1)$ times rotation around the $(J+1)$ -fold axis of α_j . The superpositions result in homogeneous density in α_j . The recurrence formula is presented using the mean value of the superposed weighed of the lattice points in a_j . The rotation angle for a_j is determined by the angle between two projected base vectors of j -dimensional space from $(j+1)$ -dimensional space.