

COMPUTER-ANIMATED POLYHEDRAL TRANSFORMATIONS

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The five regular and the thirteen semi-regular polyhedra have been known for centuries. The regular structures are known after Plato, and the semi-regular ones after Archimedes. Composed of regular faces only, these provide models of symmetry, and have inspired both artists and scientists. The drawings of Leonardo da Vinci and the graphics of Escher are among the well-known works in the visual arts. In science, the polyhedral shapes of crystals like fluorite and galena, the lattices of silicates and borates, and the protein shells of spherical viruses are well-known examples of the use of polyhedra in nature.

A central aspect in the study of structure is transformation of one structure to another. Transformations introduce the dynamic element in our classification of structures and helps us see one structure as a "state" of another. This provides the fundamental motivation for the study of transformational structures, whereby structures can change to others by adding certain elements, or changing some others. In natural structure, the study of such transforming structures becomes extremely important. Common examples of polyhedral transformations in nature are when an icosahedral viral shell undergoes a lattice transformation to release the DNA, or when the different faces of a crystal grow at different rates to produce a "transformed" polyhedron. In architecture, the use of transformational structures are useful for designing adaptive structures that respond to the changing environment.

The dynamic aspects of space structures are best realised with the use of computers. Computer-animation provides a natural medium for studying and visualising the temporal aspects of transforming structures, and provides the basis of the present collaborative work. This animation shows continually transforming polyhedra within the three polyhedral families, namely, the tetrahedral, octahedral and the icosahedral symmetries, and extends to the infinite class of prismatic symmetries. For the purposes of this presentation, the transforming polyhedra are restricted to those with mirror-symmetry; this excludes the enantiomorphs which will be presented later. The animation is an extension of the earlier computer-animation *Sketches of Polyhedra Transformations*, a 12 min. color video, by the authors with P.Hanrahan, and first premiered at the conference 'Shaping Space', Smith College, Mass. (1984).

The early sequences of the animation show the composition of the color-coded fundamental region of polyhedra, the transformations of this region to others, and the conversion of the fundamental region into the entire polyhedron by series of reflections and rotations. This is followed by sequences of the continuous pulsation of one polyhedron to another through the three families. The polyhedra are color-coded analogously within each family. The red corresponds to 3-fold, 4-fold and the 5-fold faces respectively in the tetrahedral, octahedral and the icosahedral families; the blue corresponds to the 3-fold faces, and the green is used for the 2-fold faces. The color-coding uses the RGB system for this medium, and corresponds to the red-yellow-blue in the pigment system.

The next set of sequences show the build-up of the cubic "reference space" for each family

of transforming polyhedra. The polyhedra can transform continually to one another within this reference cube where very distinct location represents a distinct polyhedron. For the purposes of illustration, a lattice of $5 \times 5 \times 5$ polyhedra are shown to display their transformation *in space*. A few rotations, a tumble and a fly-through bring a closer look at this space for each family. Transformations, *in time*, within this space are shown next by a single polyhedron moving through this space. As the polyhedron moves, it changes its shape. The correlation between the changing polyhedron and the changing reference space is shown in the last sequence.

The animation is produced at the Computer Graphics Laboratory, New York Institute of Technology, using NVIT's modelling and rendering software. Special real-time animation software was written by Patrick Hanrahan¹. The computational geometric model is based on Robert McDermott's algebraic solution for the transformations². The polyhedral transformation concepts were first described in Hareesh Lalvani's doctoral dissertation³. The animation used Vax 11/780s, was previewed on Evans and Sutherland's Multi-Picture System, and was rendered on the Ikonas. The animation is 9 minutes long at the time of this writing.

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References

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