A MATHEMATICAL APPROACH TO SYMMETRY IN THE PLANE WITH COMPUTER GRAPHICS

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The notion of symmetry is very wide, and has to do with practically all fields of the various sciences, including Social Sciences and Humanities. This talk will concentrate on the Mathematical aspects of symmetry, and will give an overview of how mathematics deals with this concept, especially with symmetric drawings in the plane. In particular, the definitions of transformations and their composition, of invariance, symmetry operations, and of symmetry groups will be exposed and explained. The talk will be illustrated by a variety of computer drawings.

What is this sense of symmetry and how can it be analyzed mathematically? This is the subject of the talk.

A linear transformation of the plane may be of such a kind that it preserves the sizes, the proportions and the angles within objects. Such transformations are known as isometrics. They form a group in the sense that they are invertible and that the composition of isometries is an isometry. They include translations, reflections, rotations and glide-reflections. If images drawn on the plane are left unchanged under the operation of a certain transformation, they are said to be *invariant under* this transformation. Each drawing on the plane has a set of isometries under which it remains invariant. This set is known as the *symmetry group* of the drawing.

Some images may be invariant only under the identity map, in which case the image has no symmetry. Otherwise they may be invariant under several isometry operations. The various types of symmetries are classified according to the transformations under which they remain invariant. There are 17 different symmetry groups in the plane and these will be constructed together with their symmetry groups and illustrations.

The different symmetry groups and illustrations are constructed by adding more and more operations under which they are invariant. The complexity of a symmetry group is determined by the number of invariant operations it contains. Note that this has nothing to do with the visual complexity of the image. On the contrary. The more symmetric an image seems, the simpler it looks, because symmetric images seem to contain some built-in order that is pleasing to the eyes.

These and other concepts will be dealt with in the lecture.