

New Models of Synergetics Topology

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All the unstable Archimedean polyhedral systems can be symmetrically transformed into platonic polyhedral systems while still maintaining the same distances between the adjacent two vertices. Furthermore, these platonic polyhedral systems can always ultimately be transformed into at least one of the three possible cases of fundamental omnitriangulated structural systems, viz. the tetrahedron, the octahedron and the icosahedron. Finally, the periodic relations inherent in these rational transformations can be reduced to "Structural Quanta".

Symmetrical Contraction of Cube with Axial Spinnability of Two Poles: Tetrahedral Progression

The axis transfixes the cube through the opposing north and south poles (1). If each pole rotates about the axis in the opposite direction by the angular closing of adjacent edges (1), the cube will contract symmetrically to bring together the two opposed vertices which lie on the diagonal of the square (2) until it becomes first the incomplete octahedral phase (3). In this case the two sets of double edges suggest polarization. Next, as the two poles approach each other on the axis to come together the other pairs of opposing vertices (4) (5), the cube folds into two congruent tetrahedra (6). The cube consists of a positive and a negative tetrahedron and is an indivisible unity. In other words, since

the total number of edges and vertices of the cube is exactly twice that of the tetrahedron, we can refer to the cube as "two quanta" in our tetrahedral system. Ranking the models in terms of the number of edges, column 5 of the table shows how 18 types of platonic and Archimedean polyhedra are all, without exception, composed of edges whose numbers are multiples of six.

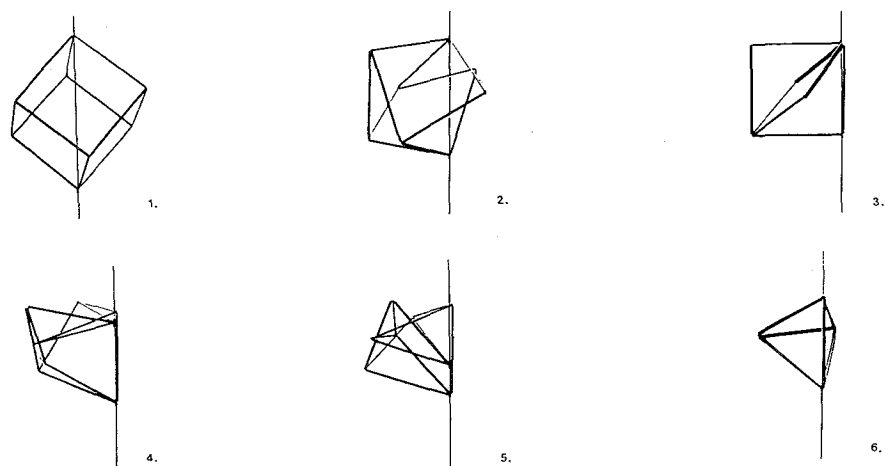


Fig. 1. Cube can be Transformable into Two Tetrahedra

Basic Frame Models of Synergetics Topology

We can thread a nylon string through each of the 12 equal length tubes twice to make a loop and fasten them together with three tubes joined at each of 8 corners to make the cube, which proves to be structurally unstable. The tubular frame models of all platonic and Archimedean polyhedra can be constructed by using this loop-ligature technique.

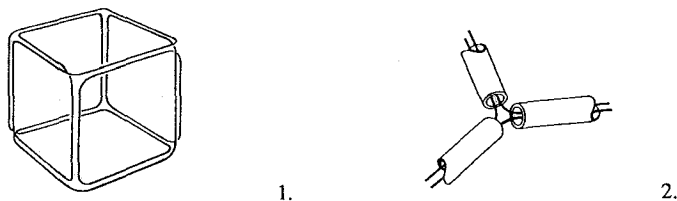


Fig. 2. Synergetics Path: A Loop Formed by Stringing Twice Through Each Tube
1. A loop of the cube. 2. Detail of the loop joint connected with three tubes.

Periodic Table of Synergetics Topology: Structural Quanta

1	2	3	4	5	6										7	8
	$3V - 6 = E$ $V = \text{Vertices}$ $E = \text{Edges}$	$3V - 6 > E$	Name of polyhedron	The number of vector-struts	The number of congruent polyhedra: Structural Quanta										Topological synergetics hierarchy	Quantum leap of first four prime numbers
					2 Archimedean polyhedra	5 Doubling Platonic polyhedra	5 Platonic polyhedra	3 Ground States								
1			TETRAHEDRON	6											1	1
2			OCTAHEDRON	12										1 → (2)	2	1
3			HEXAHEDRON	12									1	2	2	1
4			TRUNCATED TETRAHEDRON	18										3	3	1
5			CUBOCTAHEDRON	24		1								2 → 4	4	2 ²
6			ICOSAHEDRON	30									1	5	5	1
7			DODECAHEDRON	30									1	5	5	1
8			TRUNCATED OCTAHEDRON	36									3	6	2 × 3	1
9			TRUNCATED CUBE	36									3	6	2 × 3	2
10			RHOMBI-CUBOCTAHEDRON	48		2								4 → 8	8	2 ³
11			SNUB CUBE	60										5 → 10	10	2 × 5
12			ICOSI-DODECAHEDRON	60		1								5 → 10	10	2 × 5
13			TRUNCATED CUBOCTAHEDRON	72		3								6 → 12	12	2 ² × 3
14			TRUNCATED DODECAHEDRON	90										3 → 15	15	3 × 5
15			TRUNCATED DODECAHEDRON	90										3 → 15	15	3 × 5
16			RHOMBICOSI-DODECAHEDRON	120		2								10 → 20	20	2 ² × 5
17			SNUB DODECAHEDRON	150										5 → 25	25	5 ²
18			TRUNCATED ICOSI-DODECAHEDRON	180		3								6 → 30	30	2 × 3 × 5

Each of the 5 Doubling Platonic polyhedra binds two struts at each of the midpoints of its edges. Some of the transformations of Nos. 2, 6, 11, 12 and 17 require the detaching and rejoining of the vertexial connections. They are represented by means of (), i.e. irreversibility.