UNIFORM POLYHEDRA FOR BUILDING STRUCTURES

by

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General introduction

Many building structures are in their overall shape or in their internal configuration based on the geometry of one or more kinds of the so-called 'uniform' polyhedra. The cube and the prisms are very popular in this respect, but these are in fact members of a large family of forms that have great potentials for application in building. It is therefore interesting not only but highly relevant also to undertake a systematic study after the properties of this group of figures in order to find out what their characteristics are, what their binding factors and what their differences.

Definition of the uniform polyhedra [1]

1. They are composed of one or more kinds of plane, regular polygons with 3, 4, 5, 6, 8 or (maximally) 10 edges.
2. The polygons meet in pairs at a common edge.
3. The dihedral angle at such an edge is always convex (= less than 180°, if seen from the interior).
4. All vertices of a polyhedron lie on one circumscribed sphere.
5. All these vertices are identical, which means that around each vertex of a particular polyhedron the polygons are grouped in the same number, kind and order of sequence.

Different kinds of polyhedra

It is easy to understand that within these limits the minimum total number of polygons around a vertex is 3, the maximum number 5 and it is also simple to prove, that not more than 5 totally regular polyhedra can exist. These regular solids are composed out of one kind of faces each.
Polyhedra are called semi-regular if more than one kind of polygons is used for their construction. Thus a group of 30 polyhedra in total, answers the definition of uniformity (see Fig. 1):
- 5 regular or Platonic solids, consisting of 8 or 20 triangles, of 6 squares or of 12 pentagons
- 15 semi-regular or Archimedean solids (including the enantiomorphic or left-handed versions of the snub cube and the snub dodecahedron)
- 5 prisms, having two parallel congruent regular polygon faces with squares as the interconnecting sides (the square prism is identical to the cube)
- 5 antiprisms with two twisted parallel polygons and with triangles as sides (the triangular antiprism is identical to the octahedron)

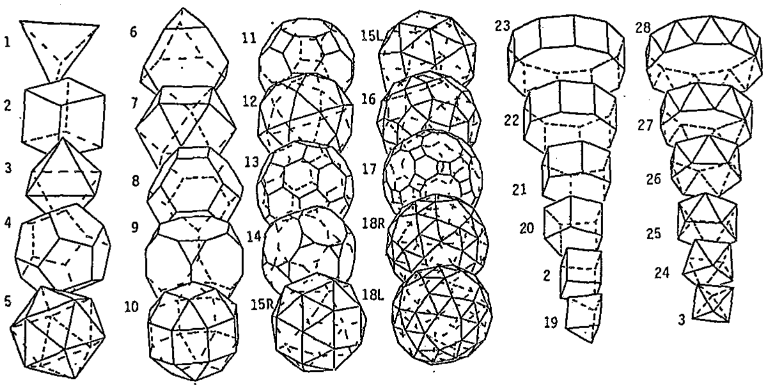

Fig. 1. *Review of the regular and semi-regular uniform polyhedra:*
A. Platonic: tetrahedron(1), cube(2), octahedron(3), dodecahedron(4), icosahedron(5)
B. Archimedean: truncated tetrahedron(6), cuboctahedron (7), truncated octahedron(8), truncated cube(9), rhombicuboctahedron(10), truncated cuboctahedron (11), icosidodecahedron(12), truncated icosahedron(13), truncated dodecahedron(14), righthanded snub cube(15R), lefthanded snub cube(15L), rhombicosidodecahedron(16), truncated icosidodecahedron(17), righthanded snub dodecahedron(18R), lefthanded snub dodecahedron(18L)
D. Prisms: trigonal(19), square(2), pentagonal(20), hexagonal(21), octagonal(22), decagonal(23)
E. Antiprisms: trigonal(3), square(24), pentagonal(25), hexagonal(26), octagonal(27), decagonal(28)

Computer technique for visual representation [3]

The coordinates and other geometric aspects of uniform polyhedra can be generated automatically with the help of computers by firstly forming each of the different kinds of polygons out of which the polyhedron consists and then reproducing it by combined translation and rotation in space as many times as it occurs in the considered polyhedron. Fig. 2. shows the principle of such a procedure as developed for study purposes on personal computers.

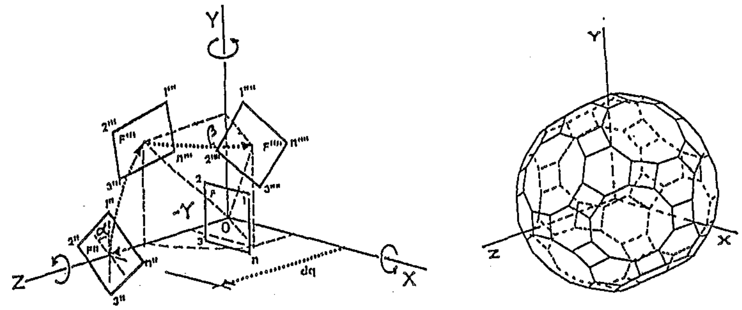

Fig. 2 *Combined translation and rotation of polygons for the formation of polyhedra*

The use of polyhedral forms in building [3]

The role of these polyhedra for the formgiving possibilities in building is very important, although this is not always recognized.

- all trivial uses of cubic and prismatic shapes (vertical or *horizontal, pure or distorted versions)*
- many solitary applications with macroforms, derived from one of the more complex polyhedra or of their reciprocal (dual) forms
- prismatic and antiprismatic folding structures
- close-packings of one or more kinds of solids in conglomerates or in space structures
- pyramidized or polar versions of solids in order to reduce the relative size of larger polygons
- basic geometry of geodesically subdivided dome structures - mainly on the basis of octahedron, icosahedron, or triacontahedron (reciprocal of icosidodecahedron)

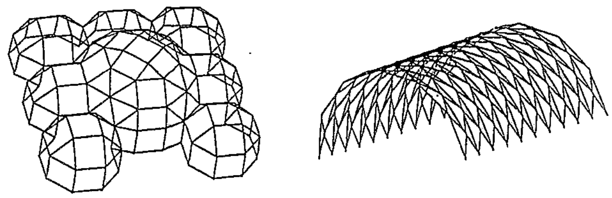

Fig. 3. *Polyhedral building and antiprismatic structure*

Tri-axial ellipsoids [2]

Polyhedra can be considered as subdivision patterns of spherical shapes. A higher break-down frequency provides a closer approximation to the sphere. If to such a dome structure different radii are given in X-, Y- and Z-direction of the coordinate system, then a tri-axial ellipsoid is obtained. A horizontal cross-section according the XY-plane (see *Fig.* 4) has the form of an ellipse. Any vertical cross-section through the Z-axis is an ellipse also, having its foot points on the horizontal ellipse. The equation of the ellipse is of the second power, but the main form of this ellipse can be influenced considerably by raising or lowering this power.(squared or flattened if respectively larger or smaller than 2, straight or hollow if respectively equal to or smaller than 1).

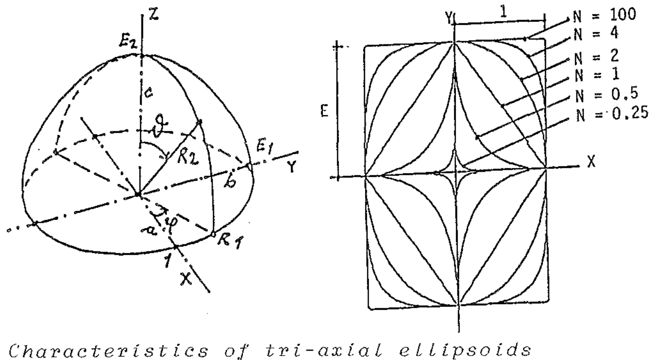

Fig. 4. *Characteristics of tri-axial ellipsoids*

The combination of ellipses of different powers leads to a great variety of visually but not basically different forms. A matrix of the principal possibilities is shown in Fig. 5, where their shapes are made visible by the superposition of grids that are generated on the basis of one of the regular polyhedra with triangular faces (the octahedron in this case).

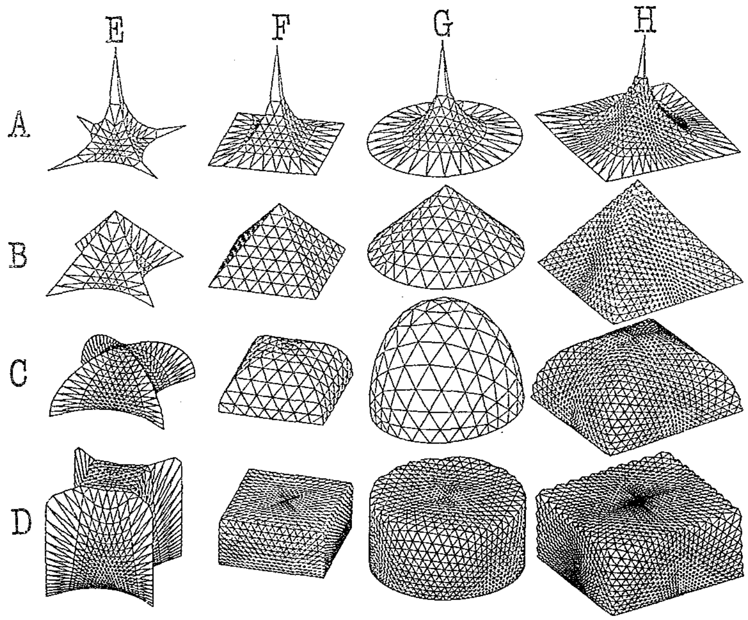

Fig. 5. *Matrix of different shapes with varying power of the ellipses (the sphere is a special case)*

Further prospects

It is of vital importance that a general approach will become available with which polyhedra and related forms can be described so that they are visible and tangible. The computer is a substantial help in reaching this ideal. Alphanumeric output of the relevant geometric data facilitate computer aided production of the construction members.

Bibliography

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[3] Huybers, P. and G. van der Ende, Polyhedral geometry as a basis for building structures and other materials systems. Chapter in "Symmetry in a Kaleidoscope", D. Nagy and G. Darvas eds., Budapest, 1989., 40 pp.