FRUSTRATION: SOURCE OF COMPLEXITY

TOHRU OGAWA

E-mail: Ogawa-t@koalanet.ne.jp

An important role of fundamental science is to prepare some views and concepts, which are available in extending the scope of science. The attempt in this paper is based on this philosophy. Cellular automata are often studied from a similar point of view, in which simple basic laws governing local processes give rise to highly complex phenomena. These studies are very important in the extension of physical science in the future even though the correspondence with the real physical world may be at most metaphorical or suggestive. For further development along this line, it is desirable to try to include some conservation laws and frustration phenomena in models of cellular automata. A frustrated system is very interesting and important from this point of view because of abundance of states and phenomena appearing in it. We may bear in mind that, in living organisms, a wide variety of phenomena are contained within a very narrow energy range. We may expect the study of a frustrated system to provide an insight into more complex systems than treated hitherto.

The cellular automaton models are usually deterministic. However it is very difficult to construct a purely deterministic model with conservation and frustration. Therefore a kinetic model containing stochastic (probabilistic) terms is studied in this paper.

The antiferromagnetic Ising model on a triangular lattice is typical of such systems. The frustration produces a degeneracy of the ground-state. The number of degrees of freedom of the ground-states is exp[0.3231] = 1.381 per site. The residual entropy of the system, which was rigorously calculated by Wannier, is proportional to the logarithm of this value. Almost nothing is known of the details of the degenerate ground-states. Some years ago, the author studied their cluster structure in relation to some maze-like patterns observed in the monodispersive latex system in a (2+e)-dimensional space.

Though the present investigation is on similar lines, its purpose is more abstract. It is the so-called "glass transition" that is the closest physical effect to the present research. It is supposed that there is no essential difference between a liquid and a glass so far as their static structures are concerned. Or, at least, no abrupt change in the static structure can be expected. The difference in structure between a liquid and a glass may be expected to lie in the dynamics of their phase spaces. If so, the present model is the simplest one that can be investigated in detail.

Reference

T. Ogawa and Y. Nakajima; "Frustration, Degeneracy, and Forms: A View of the Antiferromagnetic Ising Model on a Triangular Lattice" *Progress of Theoretical Physics*, Supplement No. 87 (1986), pp. 90-101.

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