PATTERN DESIGN BY IMPROPER USE OF MATHCAD

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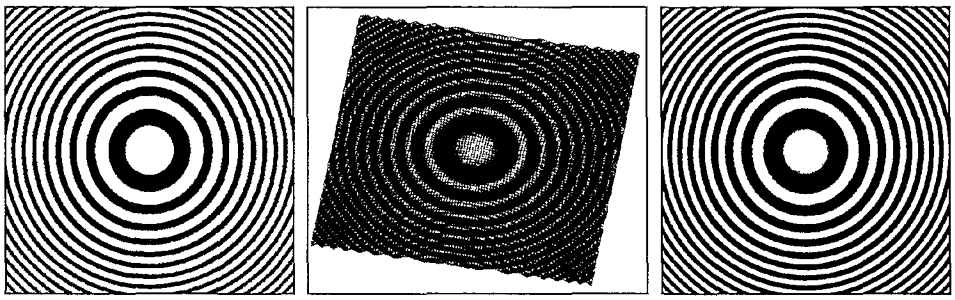
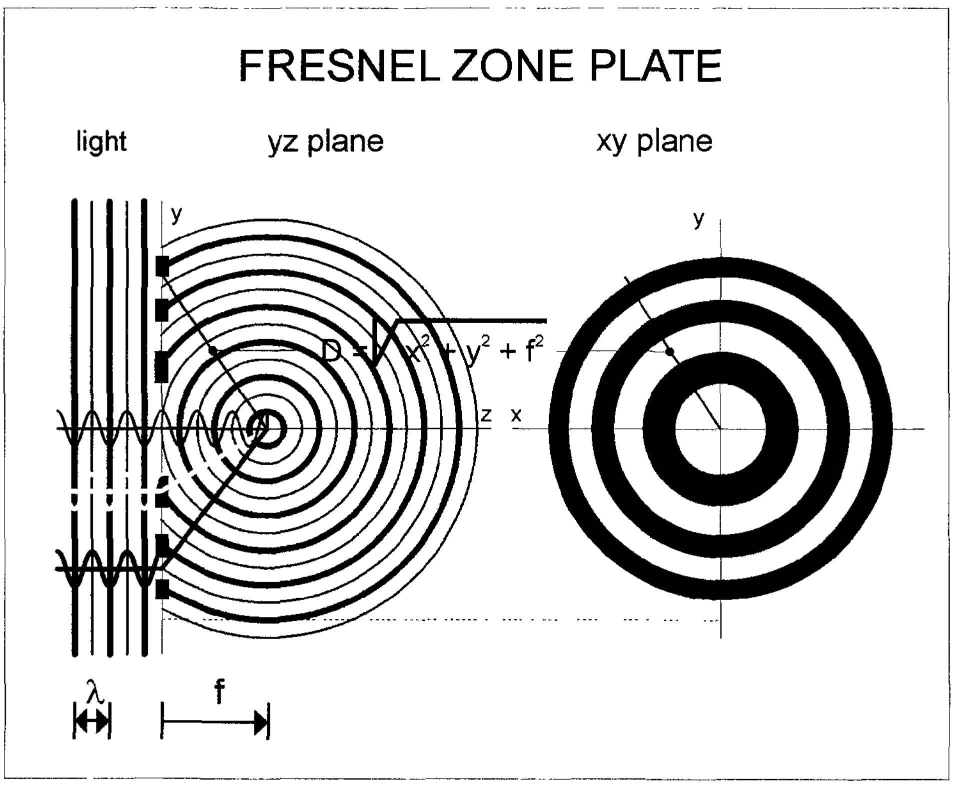
Abstract: *If one plots a function in Mathcad (or another math program) one has to define the x-values of the points to be plotted. These points are then successively joined by a line. When plotting a repetitive function, with the point spacing bigger than its repetition length, a nearly erratic plot is the result. The same can be done with contour plots. Here the contours are interpolated as well but they depend on the xy-values of the grid points. Some pattern designs extracted from the animation resulting from this improper way of plotting are shown.*

INTRODUCTION

The pattern design started from a camera obscura or pinhole camera. One of the disadvantages of this camera is the very long exposure time it needs. A solution for this problem may be the use of a zone plate. Such a plate can be plotted with any math-program, printed, photographed and reduced on microfilm to the appropriate dimension. The plotted function is shown below as a surface plot (middle) and as contour plots. The contour plot with 2 levels (left) is a Fresnel zone plate. A Gabor zone plate (right) is the result when we plot it with an infinite number of levels from white to black, resulting in continuous greyscales from high to deep.



The illustration below explains the geometrical optics of the Fresnel zone plate, based on the interference of light. The ray in black has constructive, the one in white has destructive interference in the focal point with the central ray.



In order to define the pattern for the zone plate we start from the focal point. We divide the distance D by the wavelength and express it as a cosine function to obtain the amplitude in the z direction.

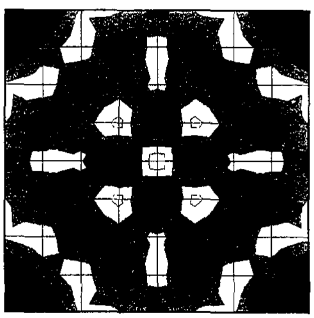
$$\text{amplitude} = \cos\left(\frac{\sqrt{x^2+y^2+f^2}}{\lambda}\cdot 2\pi\right)$$

If the function is positive, this ray has constructive interference, if not destructive interference with the central ray in the focal point. If we eliminate now the rays with destructive interference by use of black rings we have an amplification of monochromatic light in nearly the same way as with an ordinary lens (in fact things are more complicated than explained here). A Fresnel zone plate with only black and transparent rings has not a sharp focal point for a given wavelength. A Gabor zone plate corrects this.

However, a disadvantage of such a zone plate is the big chromatic aberration, as its focal distance is to a high degree depending on the wavelength. The use of a green filter gives a substantial amelioration, because green is one of the additive main colours (RGB), and these main colours give a more monochromatic light. Besides, as there is a lot of green light in normal white light, we lose less light with a green instead of a red or blue filter.

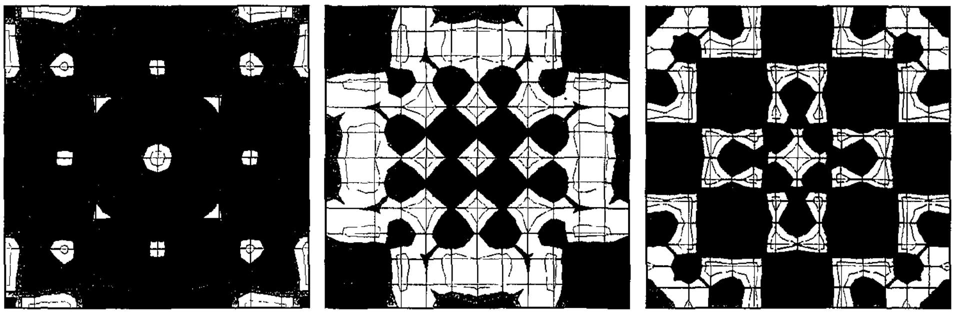
PATTERN GENERATION

The surprise at first was big when the result below, with 5 levels was obtained.



Afterwards the explanation was very simple; the contour plot above was made with a 9x9 matrix (for a "quick" result). The plot program then interpolates the contours (see the last figure in the paper). For a more correct plot a grid with much higher density was needed (and a lot more of rendering time).

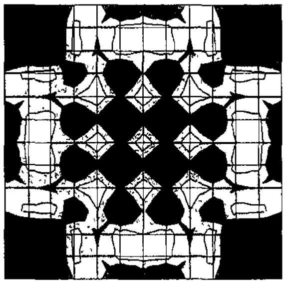
The first disillusion turned to curiosity; the lens design switched to pattern design. Experimenting with exactly the same formula, but with other grid values (11x11, 13x13 and 15x15 matrix), patterns as illustrated below are obtained.



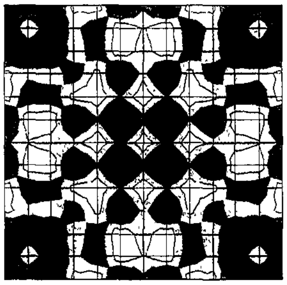
Only by changing the grid density, a nearly infinite number of different patterns can be generated from one and the same formula.

The question arises whether the patterns can continuously be transformed into each other to make animations. This is impossible by changing the number of grids, as we need integers to build the matrix for the contour plot. But gradually changing the λ-value can do the job (also "f" can do it).

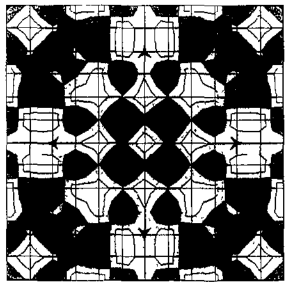
The next illustration shows such a continuous transformation of a pattern with the same grid density by changing the λ–value step by step.



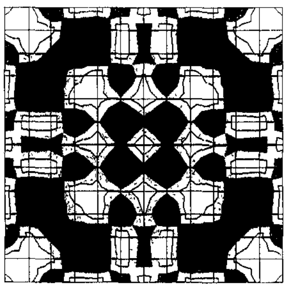
λ = 0 200



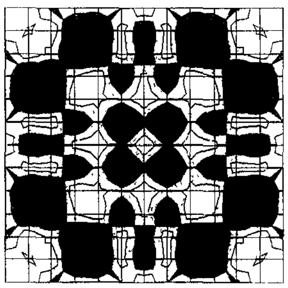
λ = 0 201



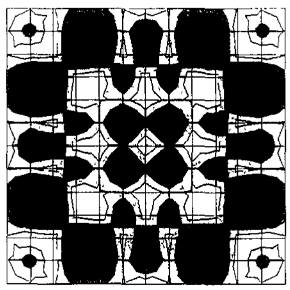
λ = 0 202



λ = 0 203

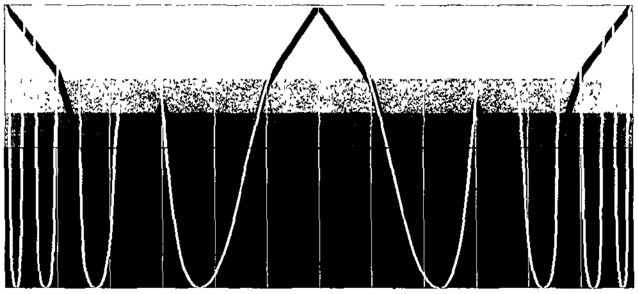


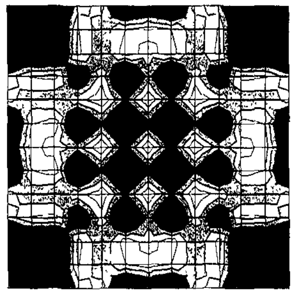
λ = 0 204



λ = 0 205

The patterns look too homogeneous. Therefore the amplitude is scaled from centre to edge. The figure below illustrates the principle of the erratic plot and the result of the scaling.

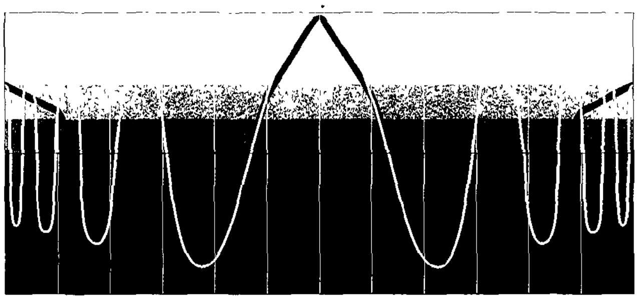


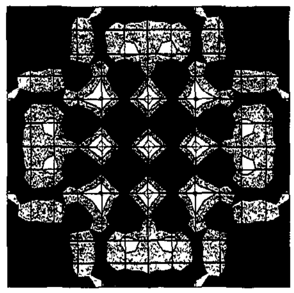


The discrete function is more correct in the centre than at the edges **Before scaling**

The intersections of the continuous function with the vertical grid give the points to fill the matrix
The contours are interpolated by the intersections of each grey shift with the discrete function

Scaling the function gives a differentiation of grey and density from centre to edge **After scaling**





After careful experimentation with different step and scaling values, the final animation "MAT⊛MIUM 2002" was made. The mini-animation on the Matomium website is made for internet use, as the original animation was 80Mb!

References

............*Mathcad7*, ©1986-97 by MathSoft, Inc., *MathSoft Kernel Maple* ©1994 by Waterloo Maple; *File Filters* ©1985-97 by Circle Systems, Inc.

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