ENDLESS FORM-WORLD GENERATED BY INTEGER PERMUTATION

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Szabadág = semmibe vett szükségszerüség? [Is freedom=ignored necessity? in Hungarian] *Müvészet*, 1978, 6
Patterns based on permutations, *Symmetry: Culture and Science*, Vol. 3 No. 2 (1992)
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Folk art and symmetries of permutation-generated signs, *Symmetry: Art and Science*, Vol. 1 No. 1-2 (2001)

Abstract: *The permutation generated patterns, which are created in square matrices, consisting of "n" rows and "n" columns are naturally arranged in tables. Selecting an "n", the number of basic forms (or signs) is n! x n!. This "world" offers really "crystal-like" symmetries, and beyond this, several of its elements we can find in the folk art of several South American, Middle American, Asian and European countries. It would be a far reaching and fruitful task of etnomathematic to make serious research-work on this field.*

1. Introduction

How to generate the above-mentioned signs? The simplest rule is: the M(c, r) n x n matrix has to be filled so that if the columns are c(1)=n, c(2)=n-1,...c(n)=1 (from left to right), the rows are r(1)=1, r(2)=2,...,r(n)=n (from up to down), and n(c) + n(r) > n, than M(c,r) = 1, otherwise it is 0. When we filled the first sign, than we take the next permutation of the rows and leave the columns intact, fill the next sign, etc. When we have no more possibility for the new permutation of the rows, keeping the r=1 at the first place (which means that we filled (n-1)! signs), then we start a new row of the signs, starting with 2 at the first place.

If we started with the n=4 and the sequence of the columns are 4,3,2,1, of the rows 1,2,3,4, then we permute the rows as 1,2,4,3; 1,3,2,4; 1,3,4,2; 1,4,2,3; and 1,4,3,2, all together 3! =6 signs. Then we cannot keep 1 at the first place, start a new row of the signs with 2,1,3,4 row-sequence, and continue with 2,1,4,3; 2,3,1,4; 2,3,4,1; 2,4,1,3; and 2,4,3,1. The next line of the signs will start with 3,1,2,4, and the last with 4,1,2,3. The 24 signs, arranged in four rows and six columns, will create the image of Fig. 1. It is the first table. We leave a gap, the size of a unit-square, between the signs, and the size of a sign between the tables.

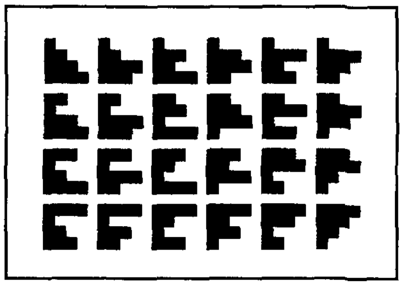

Fig.1

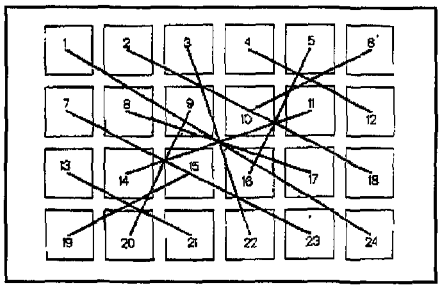

Fig. 2

After this we have to change the column sequence from 4,3,2,1 to 4,3,1,2 and start again with the row 1,2,3,4. Similarly to the previous one, we can fill the second table, etc. When we finished the sixth table (the first row of the tables), we have to start the next column-sequence with 3,4,2,1. The third row of the tables will start with 2,4,3,1 column arrangements, the fourth (the last, if n=4) with 1,4,3,2.

Finishing this, we can see that in the full block we will have n x (n-1)! number of tables with n x (n-1)! number of signs in each of them, which gives n! x n! number of signs. The number of signs in the tables (equally, the number of tables in a block) are growing so rapidly that we meet with a very interesting phenomenon: let us select n=5, and the size of the unit square=1 mm, which fits a sign in a 5 x 5 mm square. If we wish to print out the full block, then we will realize that the printout is more than 3.5 meters long. It means that if n > 5 then we cannot see the full block from one standing point. To be able to "walk around" and to see the generated form-world we need the help of a computer. We can imagine that we can move the screen around as a special window within our permutation-generated "world".

2 The basic symmetries

Using the n=4 as an example, we can find within the block as well as within the tables several symmetry connections. Fig. 2 shows these connecting lines. The famous saying "as above so below" is true here also. The basic symmetries in the block are the same ones as in the tables. Regardless of the "n", in one table there are two and only two diagonal-symmetric signs. When we investigate the signs in a table and the tables in the block, we can find symmetries, which are the same within a table and within the block. The lines are connecting those places where we can find the same diagonally symmetrical signs (in rotated positions) and in the same relative positions from these signs we will find the same surroundings in the connected tables also. These are important information about the so-called "logical places".

3 Some important signs

It is a remarkable fact that in the folk art we can find certain signs, which are connected to each other. The "stairs" (the first and last signs of the first table) many times depicted together, in Hungarian cross stitch symbols as well as in North American Hopi Indian images or on Aztec shield and woven materials from Peru, with the "spirals". We can find these forms together even as a clay vessel in Peru. Fig. 3 shows a Hopi-Indian sign and Fig. 4 are Hungarian cross-stitch elements from the region of the river Tisza.

It is remarkable, that on this Hopi sign we can see two different types of spirals beside the stairs.

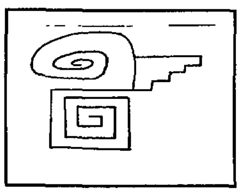

Fig. 3

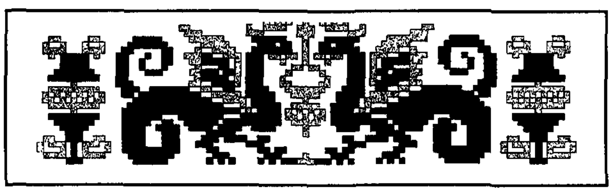

Fig. 4

4 Symmetry of symmetries

In the folk art we can find Hanti shirts, which shows elements of the same "logical places" when "n" = 3, 4, and 5, and these are on the same item. (See Fig. 5). Naturally, it would be a rather silly idea to suppose that the maker of this item had any idea about a sign system. The elements are coming from the folk soul. However, it is really interesting that on the stole we can see the "n"=4 system 11th table 3rd sign and under the "V" (which is at the end of the stole) we can find the "n"=5 version of it.

Every country or area has some characteristic patterns, which they use as their form-elements. It would be an interesting task of etnomathematic to investigate these patterns, and find out, why can we find certain sign-combinations (e.g. stairs and spirals together) in different areas of the world, thousands of kilometers from each other.

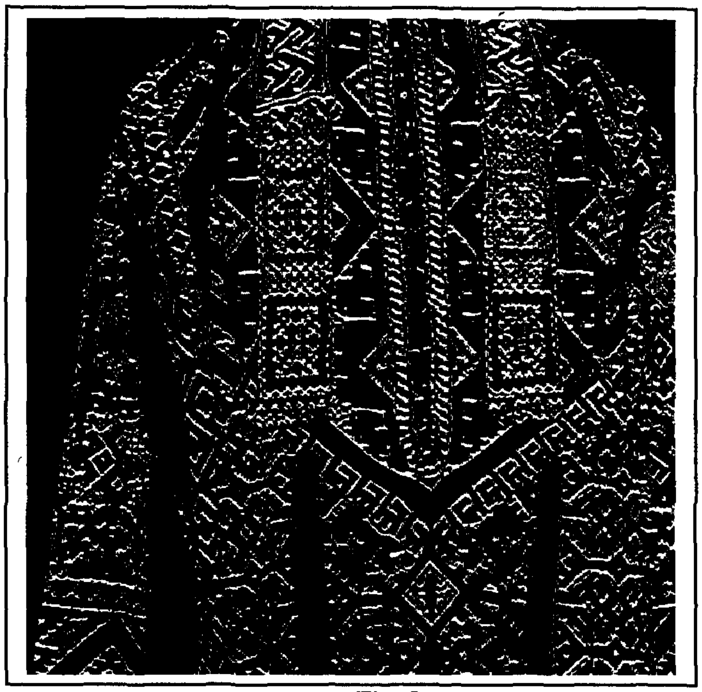

Fig. 5

The best way to see, understand and enjoy the symmetry-world of this endless and amazing system is to "step into it" and create the signs ourselves. It can strongly improve our form-recognition also.

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