

How Plato designed Atlantis



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Fields of interest: geometry and architecture in antiquity, Neoplatonism, metrological systems of Ancient Egypt, Greece and the Roman Empire

Some recent achievements: presenter at Mathematics 2000 Festival, University of Melbourne; divisional winner in Australian short story competition (2000)

Remarks on references and abbreviations

The works of classical writers often have ciphers printed in the margins of the translations to indicate a standard means of precise reference to passages. For example, in the case of Plato they indicate the pages in the edition of the philosopher's works by Stephanus (Henri Estienne), Geneva, 1578. To accord with tradition, references to Plato's writings are presented in this manner along with references to more accessible popular versions of his books. Hence, in the text of this exposition, references to classical works are presented in the following way:

Plato.

Lee, D. *Timaeus* and *Critias*, p. 137/S114

S114 indicates the page in the Stephanus edition.

Herodotus. In the case of Herodotus (*The Histories*), the numbering system is that used by J. Marincola from the de Sélincourt translation, e.g.,

Marincola, p. 88/H2.9

H means Herodotus and 2.9 indicates Book Two, passage nine.

Vitruvius. For Vitruvius (*The Ten Books on Architecture*):

Morgan, p. 27/V1.6.9

V means Vitruvius and 1.6.9 indicates Book One, chapter 6, paragraph 9.

Plutarch. Lastly, for Plutarch (*Moralia V*),

Babbitt, pp. 175–7/S381

S381 indicates page 381 in the *Books of the Moralia*, the edition of Stephanus, 1572.

Sometimes, for clarity, the letters BI are used to distinguish British imperial measures from ancient Greek and Roman measures: for example, 607.5 BI feet.

SPECIAL NOTES

(1) Historians of metrology have noted the use of a range of “feet” of varying values in ancient Greece. In all important cases the present writer is able to demonstrate they are proportionally related in a specific geometric configuration to the so-called Greek/Parthenon foot, the ancient Egyptian remen and the ancient Egyptian cubits. The matter is comprehensively dealt with in a new paper tentatively entitled *The Creation of Measure in Antiquity* now in preparation (February 2003). The only Greek measures discussed at length in this account are the so-called Parthenon foot of about 12.15 inches or 308.6 mm, the Parthenon cubit (equal to $1\frac{1}{2}$ Parthenon feet), and the stade of 600 Parthenon feet. As an example of a proportional relationship, the so-called Doric foot is the Parthenon foot multiplied by $\frac{36}{35}$ twice. This measure, about 326.5 mm, had a special design function, as did the $\frac{36}{35}$ proportion; they are explicated in the new paper.

(2) Since the *Matomium* Conference in Brussels in April 2002, when this paper was first presented, the measures in the design system described herein have been discovered, inventively encrypted, in a famous classical Roman text. Ancient Egyptian, Greek and Roman measures have been expressed in the text in terms of the so-called British imperial inch, with the values exactly the same as shown in this exposition because the same geometry was used. This far-reaching discovery (February 2003) is presently being incorporated into *The Creation of Measure in Antiquity*.

How Plato designed Atlantis

1. Preamble

1.1. The material and the sources.

This exposition is extracted from a larger work entitled *How Plato designed Atlantis and where Vitruvius obtained his model for Man*. The two subjects are combined in the larger exposition because the designs are linked. In fact, in the larger work it is demonstrated that Vitruvius (fl. 1st century BC), author of *The Ten Books on Architecture* and an admirer of Plato (Morgan, p. 195/Introduction to Book Seven), has reformulated the Greek philosopher's design for Atlantis in several ingenious ways, one of which is the mathematical design for a "well shaped man". The larger exposition, though, contains more evidence to prove the case on Atlantis than is presented here. That said, there is sufficient evidence in this version to justify the title's contention.

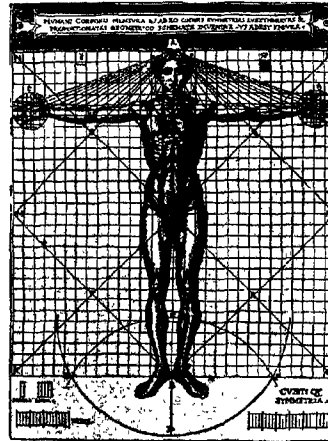


Figure 1: Vitruvius's "well shaped man", as realized by Cesar Cesariano in his edition of Vitruvius (1521).

1.2. Plato.

The philosopher Plato, 427–347 BC, a disciple of Socrates, is a seminal figure in the Western intellectual tradition. Two of his late works *Timaeus* and the continuation *Critias* contain the earliest accounts of the mythical lost continent of Atlantis. Plato says the information about Atlantis was given to Solon, an Athenian and “the wisest of the wise men”, by an Egyptian priest (Lee, *Timaeus* and *Critias*, p. 33–7/S20–5). The principal physical description of the country is contained in *Critias*, which has been described by Desmond Lee, a translator of the books, as the “first work of science fiction”. The most detailed description of the place given by the philosopher, though, is of the capital city, in particular the inner citadel, a circular island surrounded by a concentric arrangement of rings of water and land. Amongst other things, the inspiring source of the design concept of the overall layout of the land is unveiled here along with its encrypted mathematical nature.

1.3. Plato and time measurement.

In the Introduction to *Timaeus* and *Critias* Lee writes:

Plato was aware of the close connection between time and time measurement. Can we speak of one without the other? And if not, are we not bound to say that ‘time came into being with the heavens’, that is, that time in the sense we use it cannot be conceived without the instruments, processes, and movements by which we measure it? (Lee, *Timaeus* and *Critias*, p. 11)

The quote is from the *Timaeus*. The following is the passage from which it was extracted.

So time came into being with the heavens in order that, having come into being together, they should also be dissolved together if ever they are dissolved; and it was made as like as possible to eternity, which was its model. For the model exists eternally and the copy correspondingly has been and is and will be throughout the whole extent of time. (Lee, *Timaeus* and *Critias*, p. 52/S38)

It will be demonstrated that the nature of time, arising from unexpected and surprising geometric sources, is inseparable from the nature of Atlantis.

1.4. Plato and *The Laws*.

That twelve is a key time number is beyond dispute. In *The Laws*, believed to be Plato’s last major work, the philosopher sets out his ideas for the ideal state. This is his description of the ideal city:

After this, the legislator's first job is to locate the city as precisely as possible in the center of the country, provided that the site he chooses is a convenient one for a city in all other respects too (these are details which can be understood and specified easily enough). Next, he must divide the country into twelve sections. But first he ought to reserve a sacred area for Hestia, Zeus and Athena (calling it the 'acropolis'), and enclose its boundaries; he will then divide the city itself and the whole country into twelve sections by lines radiating from this central point. The twelve sections should be made equal in the sense that a section should be smaller if the soil is good, bigger if it is poor. The legislator must then mark out five thousand and forty holdings, and further divide each into two parts; he should then make an individual holding consist of two such parts coupled so that each has a partner near the center or the boundary of the state as the case may be. ... He should also divide the population into twelve sections, and arrange to distribute among them as equally as possible all wealth over and above the actual holdings (a comprehensive list will be compiled). Finally, they must allocate the sections as twelve 'holdings' for the twelve [Olympian] gods, consecrate each section to the particular god which it has drawn by lot, name it after him and call it a 'tribe'. Again, they must divide the city into twelve sections in the same way as they divided the rest of the country; and each man should be allotted two houses, one near the center of the state, one near the boundary. That will finish off the job of getting the state founded. (Saunders, pp. 215–6/S745)

Plato follows this up by stating:

Now that we have decided to divide the citizens into twelve sections, we should try to realize (after all, it's clear enough) the enormous number of divisors the subdivisions of each section have, and reflect how these in turn can be further subdivided and subdivided again until you get to 5040. This is the mathematical framework, which will yield you your brotherhoods, local administrative units, villages, your military companies and marching-columns, as well as units of coinage, liquid and dry measures, and weights. The law must regulate all these details so that the proper proportions and correspondences are observed. (Saunders, pp. 217–8/S746)

1.4.1. In a commentary on Plato's most famous work the *Republic*, James Adam, author of *The Republic of Plato* writes:

We know from the *Laws* that Plato counted 360 'days' in the year. (Adam, p. 301)

Adam's footnote to this passage states:

The number of Senators in the *Laws* is 360: these are to be divided into 12 sections of 30 each, and each section is to administer the State for one month. The number 60 with its multiples and divisors is the dominant number throughout the *Laws*. 360 'days' is of course only an ideal division of the year: see § 6. Plato elsewhere recognizes (with Philolaus) 364 ½ days (*Rep.* IX 587 E ...). (Adam, p. 301)

The number 364 ½ is half 729 which is 27 squared. The number 27 makes several notable appearances in this exposition.

1.5. Plato and the Pythagoreans.

In the Translator's Introduction to Plato's most famous work, the *Republic*, Lee remarks:

In 388–387 BC Plato visited South Italy, perhaps in order to make the acquaintance of some of the Pythagorean philosophers living there. The Orphic-Pythagorean belief in the after-life and the Pythagorean emphasis on mathematics as a philosophic discipline certainly influenced him strongly, as can be seen in the *Republic*. (Lee, *Republic*, p. xvii)

Given Plato's familiarity with Pythagorean ideas there can be no doubt that he was familiar with the well-known symbol of the Pythagoreans: the pentagram.

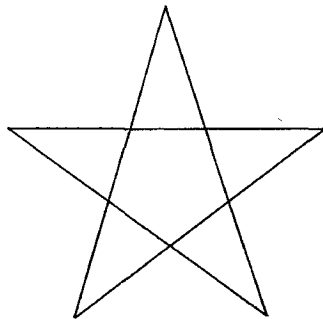


Figure 2: the pentagram, the symbol of the Pythagoreans.

2. Central Atlantis

2.1. Plato's description.

The philosopher's description of the citadel and ring arrangement in Atlantis is as follows:

The largest of the rings, to which there was access from the sea, was three stades in breadth and the ring of land within it the same. Of the second pair, the ring of water was two stades in breadth, and the ring of land again equal to it, while the ring of water running immediately around the central island was a stade across. The diameter of the island on which the palace was situated was five stades. (Lee, *Timaeus* and *Critias*, p 139/S115)

The full ring arrangement measures 27 stades in diameter: see figure 3 below. An ancient Greek stade measured around 607.5 BI feet (based on the so-called "Parthenon" foot of about 12.15 inches). The stade and the Greek/Parthenon foot are discussed later.

Lee makes the following points in his commentary on Atlantis (Lee, *Timaeus* and *Critias*, p. 152):

- a) "But what Plato is most interested in and spends most time describing is the capital city itself, and more particularly its inner citadel."
- b) "The inner citadel is shown as a series of small rings at the center, ... Its basic form was determined by Poseidon who ringed the small hill where Cleito lived with two rings of land and three of water; but its equipment and buildings are the work of the inhabitants."
- c) "The breadth of the rings of land and water is $3 + 3$, $2 + 2$, and one stade, and the central island is 5 stades across. (Is it significant that this gives a total of $27 = 3^3$?)"

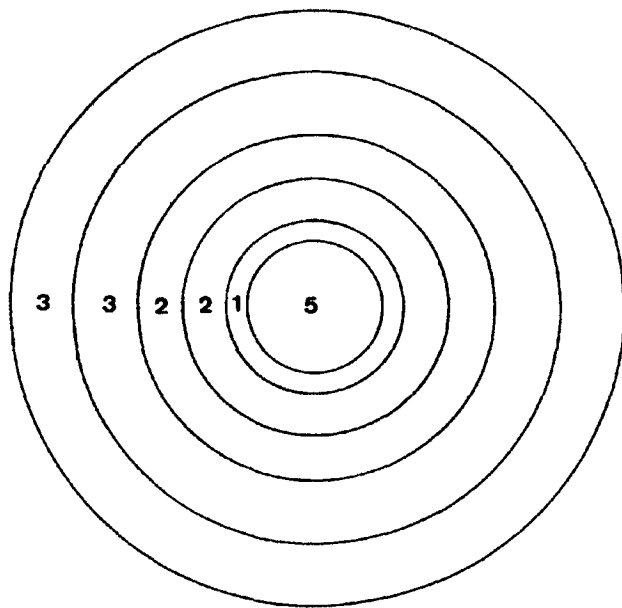


Figure 3: the rings of water and land around the central island of Atlantis five stades in diameter ($3 + 3 + 2 + 2 + 1 + 5 + 1 + 2 + 2 + 3 + 3 = 27$ stades, the diameter of the full ring arrangement).

2.1.1. Plato provides clues on the original design source for the illustration in figure 3.

He [Poseidon] begot five pairs of male twins, brought them up, and divided the island of Atlantis into ten parts, which he distributed between them. ... The eldest, the King, he gave a name from which the whole island and surrounding ocean took their designation of 'Atlantic', deriving it from Atlas the first King. His twin, to whom was allocated the furthest part of the island towards the Pillars of Heracles and facing the district now called Gadeira, was called in Greek Eumelus (Lee, *Timaeus* and *Critias*, p. 137/S113–4)

Attention is drawn to four matters: the numbers five (notably the five-stade diameter of the central island) and ten, how the name Atlantis is derived from Atlas, and the mention of Heracles (Hercules). Atlas is frequently depicted in art bearing the globe of the cosmos or the world on his shoulders.



Figure 4: statue of Atlas in Collins Street, Melbourne.

3. Dissecting the Pythagorean symbol

3.1. Five and ten.

In the Pythagorean symbol in figure 5 below—a five-pointed “star”—references to five and ten can clearly be seen

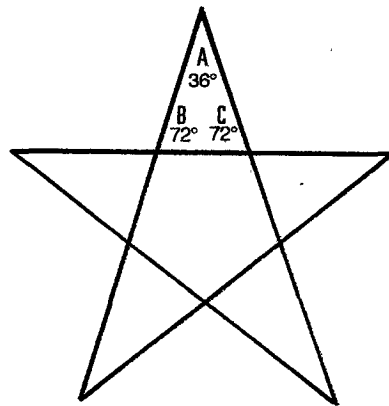


Figure 5: a five-pointed star, the pentagram.

ABC above, one of five isosceles triangles, is a one-tenth segment of a decagon (see figure 6 below). Angle A is 36° , angles B and C each 72°

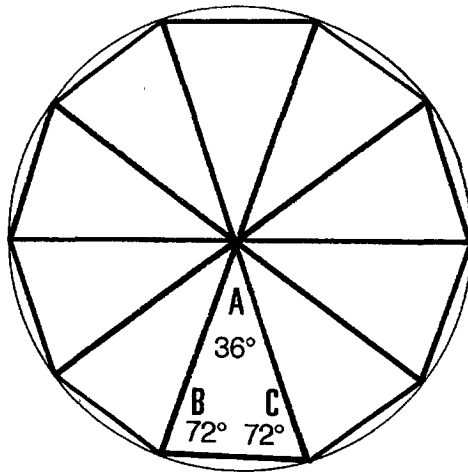


Figure 6: a decagon inside a circle.

3.2. Angles.

In the pentagram illustrated below the following angles can be discerned:

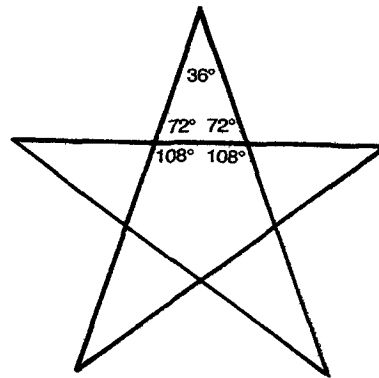


Figure 7: angles in a pentagram.

Note that 36 is 6 squared, $72 + 72$ (144) is 12 squared, and $108 + 108$ (216) is 6 cubed.

3.2.1. If 36° is taken as base one then figure 7 above can be illustrated thus:

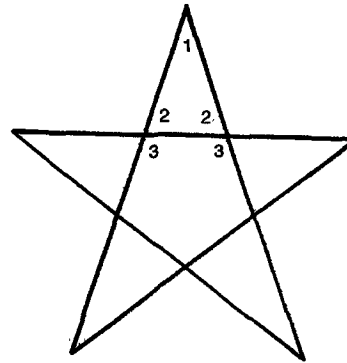


Figure 8.

Instantly the source of Plato's $3 + 3 + 2 + 2 + 1$ ring arrangement becomes apparent: see figure 3 and the preceding description by Plato of the ring arrangement in 2.1.

3.3. Time and the world.

What made Plato, the Pythagoreans and others so interested in the pentagram? An examination of the nature of time shows why.

A day contains 24 hours. Each hour contains 60 minutes and a minute contains 60 seconds. An hour contains 3600 seconds, twelve hours 43,200 seconds and a full 24-hour period, 86,400 seconds.

In antiquity, the 24-hour period was divided into twelve daylight hours and twelve nighttime hours. The latter arrangement has an ancient Egyptian origin.

The Egyptians were the first to divide the day into 24 hours; there were twelve hours of the day and twelve hours of the night. (Gardiner, p. 206)

3.3.1. In figure 9 below it can be seen that from point A to B the sum of the angles is 432° . A to C also adds up to 432° . A to B plus A to C sum to 864° . As stated, twelve hours contains 43,200 seconds and 24 hours contains 86,400 seconds.

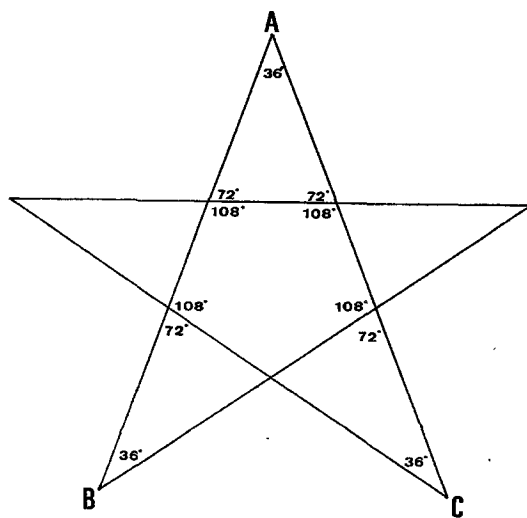


Figure 9: (A to B or A to C) $36^\circ + 72^\circ + 108^\circ + 108^\circ + 72^\circ + 36^\circ = 432^\circ$.

Furthermore, the sum of the angles in the pentagon component of the pentagram is 540° ($108^\circ \times 5$) and the sum of the angles in the five isosceles triangles is 900° ($180^\circ \times 5$): $540^\circ + 900^\circ = 1440^\circ$. A day contains 1440 minutes.

3.4. Connecting to ancient Egypt.

In ancient Egypt, right back in the pyramid age, ancient Egyptians used the five-pointed hieroglyph illustrated below to represent, amongst other things, time.



Figure 10: the five-pointed "star" hieroglyph.

The following information is from Sir Alan Gardiner's opus, *Egyptian Grammar*. It has been extracted from a description of the "star" hieroglyph pictured above, which is N14 in his Sign-list.

ideo or det. [ideogram or determinative] in 'star', 'teach', time as indicated by stars, 'month', 'hour'. (Gardiner, p. 487)

An important and far-reaching deduction can be made. As no star in the sky has ever looked anything like the five-pointed hieroglyph above, this geometric figure, abstracted from a pentagram (see figure 11 below) must have been chosen in antiquity as a symbol for time because it had an appropriate mathematical constitution. It is relevant to point out that an ancient Egyptian month of thirty days contained 720 hours, that is, 360 daytime and 360 night-time hours: cf. the angles in figure 5. Drawing lines from the apexes of the triangles to the center of the pentagon can also create the "star" hieroglyph.

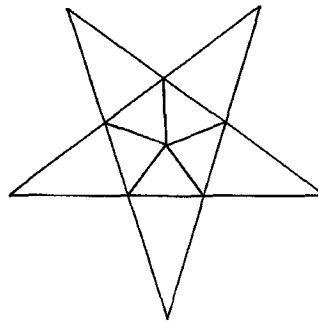


Figure 11: the "star" in the pentagram.

3.5. Eratosthenes and his measure of the world.

Eratosthenes, who became head of the Alexandrian Library around 240 BC, is credited with producing the earliest sensible measure of the world: 252,000 Greek stades. Vitruvius tells us of the matter in *The Ten Books on Architecture* (Morgan, p. 27/V1.6.9). Curiously, a few paragraphs after discussing the measure of the world the architect remarks:

Some people do indeed say that Eratosthenes could not have inferred the true measure of the earth. (Morgan, p. 28/V1.6.11)

There is a reason for this comment. Vitruvius, Eratosthenes, Plato, Plutarch (ca. 45–120 AD), Herodotus (ca. 490–420 BC) and other notable historical personages knew the real measure of the world was 216,000 (60 x 60 x 60) stades. It was a designed measure, just like the French decided a few hundred years ago that the world would have a circumference of 40,000 kilometers, that is, 40 million meters. In antiquity, the real figure was disguised from enemies and the uninitiated. Proof of the matter follows.

3.5.1. The pentagram is further examined. The lettering in the diagram below is based on that in figure 5 with the addition of letters X, D, and E. X, like B, signifies an angle of 72° . D and E are each 108° .

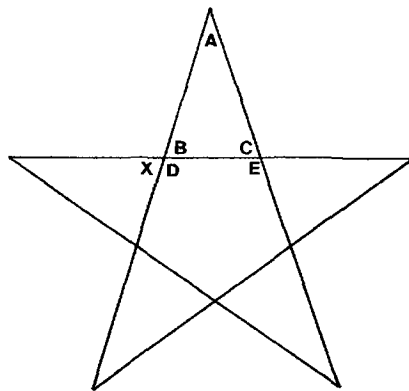


Figure 12.

The sum of $B\ 72^\circ + D\ 108^\circ + X\ 72^\circ$ is 252° . The sum of $A\ 36^\circ + B\ 72^\circ + D\ 108^\circ$ is 216° . (So is the sum of $D\ 108^\circ + E\ 108^\circ$.) The fake and the real measures, in reduced form, are therefore located in one geometric figure—and they are part of each other. A remarkable analogue of this is presented later. In the larger version of this exposition, it is demonstrated that both the fake and the real measures of the world are encrypted in the mathematical constitution of Vitruvian Man.

The Greek (sometimes called the Periclean or Parthenon) foot is estimated to have measured around 12.15 inches (Zupko, p. 6 and Petrie, *MW*, p. 5). As a stade contained 600 Greek feet, the stade was estimated to be around **7290 inches** or 607.5 feet (Zupko, p. 6).¹ Multiply the latter number by 216,000 and the product is 131,220,000 feet which is $24,852 \frac{5}{11}$ miles. The modern polar circumference measure of the world is 24,859.82 miles. Most contemporary encyclopedias provide this information. The difference is about 7.5 miles or about 12.1 km.

The Parthenon foot is readily found. It is generally agreed that the top step of the Parthenon's base was designed to have a length of 225 Greek feet (about 2735 inches) and a breadth of 100 Greek feet (about 1215 inches). Contemporary measurements (*Encyclopaedia Britannica*, Vol. 9, p. 173) show the length of the top step to be 2737.7 inches (228.14 BI feet/69.54 m) and the breadth to be 1216 inches (101.34 BI feet/30.89 m). The rate of accuracy for the layout is better than 99.9%

Some remarks about the Great Pyramid are pertinent. The base of the structure was originally 440 royal cubits square and the height 280 royal cubits (Petrie, *TPTG*, p. 183). In *Number and Divinity in Antiquity*, another exposition by the present writer, it is demonstrated that the chief design particulars for every major ancient Egyptian pyramid stem from one ingeniously contrived geometric configuration — even the layout of the three main pyramids at Giza. The designs for the pyramids and the Giza layout are a spectacular example (perhaps the greatest) of Vitruvius's maxim on symmetry and proportion:

The design of a temple depends on symmetry, the principles of which must be carefully observed by the architect. They are due to proportion, in Greek ἀναλογία. Proportion is a correspondence among the measures of the members of an entire work, and of the whole to a certain part selected as a standard. From this result the principles of symmetry. Without symmetry and proportion, there can be no principles in the design of any temple, that is, if there is no precise relation between its members, as in the case of those of a well-shaped man. (Morgan, pp. 72/V3 1 1)

The royal cubit is estimated to have measured around 20.62 inches, about 52.375 cm (Petrie, *TPTG*, p. 139 and Klein, p. 59). If the height of the Great Pyramid, 280 royal cubits, were divided by the time-related number 8.64 (refer 3.3. and 3.3 1.) the quotient would be $32 \frac{11}{27}$ royal cubits, which is about 668.24 inches. W. M. F. Petrie recorded the height of the entrance to the structure as being about 668.2 inches above the level of pavement of the pyramid (Petrie, *TPTG*, p. 55). Note that 55 Greek feet of 12.15 inches is equal to 668.25 inches. There is a 297:175 proportional relationship between the royal cubit and the Greek/Parthenon foot (proved later). The two measures are

generated in a remarkable way in the geometric configuration mentioned above, along with the well-known 22:7 proportion.

The number 8.64 is also a prominent mathematical feature in the Atlantis design and other designs that are discussed later.

3.5.2. The stade measure can be readily detected in ancient Egypt. In an article *A Ground Plan at Giza*, British researcher John Legon, using the measurements of the renowned archaeologist W. M. F. Petrie, observed the appearance of a design strategy in the rectangular layout of the three main pyramids at Giza. Legon demonstrated that the axial distance from the west side of the pyramid of Menkaure (Mycerinus) to the west side of the pyramid of Khafre (Khephren), the second largest of the pyramids, was **7289.5** inches (Legon, *AGPG*, p. 40). Compare this with the estimated measure of the stade above, 7290 inches; there is a near perfect fit. Note that a stade is equivalent to 500 ancient Egyptian remen, which can be expressed as 10,000 remen digits. Zupko and others have recorded the remen digit as measuring 0.729 inch and equal to the Roman digit (Zupko, p. 6 and Klein, p. 71). Petrie also noted the link between ancient Egyptian, Greek and Roman measures (Petrie, *MW*, p. 5). An examination is now made of the way Vitruvius, Plutarch and Herodotus referred in cryptic form to the real measure of the world

3.6. Vitruvius.

The following information is from *The Ten Books on Architecture*:

a) In the Introduction to Book Five Vitruvius says:

Then again, Pythagoras and those who came after him in his school thought it proper to employ the principles of the cube in composing books on their doctrines, and, having determined that the cube consisted of 216 lines, held that there should be no more than three cubes in any one treatise. ... The Pythagoreans appear to have drawn their analogy from the cube, because the number of lines mentioned will be fixed firmly and steadily in the memory when they have settled down, like a cube, upon a man's understanding. (Morgan, p. 130)

It can be readily guessed where the Pythagoreans obtained the number of lines from—nowhere else but from the measure of the world. (That said, there are astonishing geometric reasons as well. They are explicated in the larger version of this exposition and in *Number and Divinity in Antiquity* where more detailed accounts of the geometric knowledge of Plato and the Pythagoreans are given.)

b) The 600 Greek-foot stade was equivalent to 625 (25 squared) Roman feet. A Roman mile contained 5000 Roman feet, which was equivalent to 8 (2 cubed) stades (Zupko, p. 6). Consequently, the ancient world measure can be expressed as 27,000 (30 cubed) Roman miles. Vitruvius tells us in a discussion of aqueducts, wells and cisterns that:

It is not ineffectual to build reservoirs at intervals of 24,000 [Roman] feet, ...
(Morgan, p. 245/V8.6.7)

Twenty-four thousand Roman feet was equal to 4.8 Roman miles. This was a mathematical formulation for those in the know, and possibly a conundrum for initiates. The quotient of the world measure 27,000 Roman miles (equal to 135,000,000 Roman feet) divided by 4.8 Roman miles is 5625, which is 75 squared. Other instances of such mathematical formulations can be found in *The Ten Books on Architecture*. Here is another, again related to the measure of time.

c) In a discussion on hoisting machines, Vitruvius reports:

In our own times, however, when the pedestal of the colossal Apollo in his temple had cracked with age, they were afraid that the statue would fall and be broken, and so they contracted for the cutting of a pedestal from the same quarries. The contract was taken by one Paconius. This pedestal was twelve feet long, eight feet wide and six feet high. (Morgan, p. 289/V10.2.13)

The volume of the pedestal is 576 cubic feet: 576 is the time-number 24 squared. Apollo returns to the story later.

3.7. Plutarch.

E. A. Wallis Budge, the renowned Keeper of the Egyptian and Assyrian Antiquities in the British Museum, wrote that the ancient Egyptian god Thoth, usually depicted as a man with the head of an ibis, was, amongst other things,

“... he who reckons in heaven, the counter of the stars, the enumerator of the earth and of what is therein, and the measurer of the earth,” (Budge, Vol. 1, p. 400)

Plutarch, apart from being a teacher and a philosopher, was also a Delphic priest. (There was a famous Oracle at Delphi that was located in the temple of Apollo. The Oracle, seated on a tripod, went into a trance before she spoke.) Plutarch writes of the ibis in *Isis and Osiris*:

The strictest of the [Egyptian] priests take their lustral water for purification from a place where the ibis has drunk: for she does not drink water if it is unwholesome or tainted, nor will she approach it. By the spreading of her feet, in their relation to each other and to her bill, she makes an equilateral triangle. (Babbitt, p. 175–7/S381)

Here is an inventive reference to the ancient measure of the world. An equilateral triangle, which the feet of a tripod also form, has a particularly suitable mathematical property that connects to the ancient measure of the world: each internal angle is 60° and there are three of them.

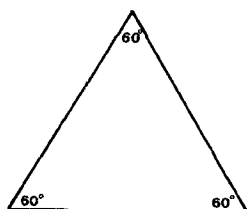


Figure 13: an equilateral triangle

And, as discussed many times already, $60 \times 60 \times 60 = 216,000$. Why Thoth was the counter of stars and the enumerator and measurer of the earth is no longer a mystery, thanks to Plutarch.

3.8. Herodotus.

Three striking examples from the writings of Herodotus provide more evidence. Book Two in *The Histories*, which deals with ancient Egypt, is the information source

3.8.1. Herodotus tells us that the coastline of Egypt is 60 *shoeni* long, and that a *schoenus* (an Egyptian measure, he says) is equal to 60 stades. The coastline is thus 3600 stades long (Marincola, p. 88/H2.6–7). Clearly, the historian has designed the measure to be exactly one-sixtieth of the ancient measure of the world, 216,000 stades which is equivalent to 3600 (60 squared) *schoeni*. Everything is in proportion.

3.8.2. Herodotus also reports:

From Heliopolis to Thebes is a nine days' voyage up the Nile, a distance of eight-one *schoeni* or 4860 *stades*. (Marincola, p. 89/H2.9)

The number nine is 3 squared and eighty-one is 9 squared. If the measure of the world is divided by 4860 stades the quotient is $44 \frac{4}{9}$ (44.444...), which is $6 \frac{2}{3}$ (6 666...)

squared. Later, the geometric source of this formulation is unveiled.

Note that nine days contains **216** (6 cubed) hours. This and the material in 3.8.1. provide compelling evidence, indeed, about the reality of the ancient world measure of 216,000 stades.

3.8.3. The famous historian tells us about the city of Buto “at the Sebennytic mouth of the Nile”:

The city also contains two other temples, one of Apollo, the other of Artemis. The shrine of Leto [mother of the two gods], where the oracle is, is a building of great size with a gateway sixty feet high, but the most remarkable sight it has to offer is not the temple itself, but a *small shrine within the enclosure made out of a single block of stone; it is cubical in shape, each side sixty feet long and sixty high.* (Marincola, p. 144/H2.155)

The purpose of the shrine is clear: the cube’s volume is 216,000 cubic feet and thus it memorializes the number associated with the ancient measure of the world.

Leto, as a matter of interest, suffered terrible birth pangs for nine days and nights, that is, 216 hours, before giving birth to Apollo (Grant, p. 118).

3.8.4. One further mention of Apollo.

Robert Graves tells us that the name has two possible meanings: “destroyer” or “apple-man” (Graves, Vol. 1, p. 57 also Vol. 2, p. 381). Apples are discussed shortly.

3.8.5. Ancient Egypt provides more proof.

E. A. Wallis Budge notes in a translation from an ancient Egyptian text that:

Soon after his marriage with Thi, Amen-hetep [Amenhotep] III. dug, in his wife’s city of Tcharu, a lake, . . . “its length 3600 cubits, its breadth 600 cubits.” (Budge, Vol. 2, p. 70)

The surface area of the lake dug by Amenhotep III (c. 1391–1353 BC) is consequently 2,160,000 square cubits. The significance of the number is obvious and it provides further evidence that the measure of the world has been around for a very long time indeed. In this regard, recall the discussion in 3.5.2. of the manifestation of the stade measure in the Giza, pyramid layout.

4. In and around Atlantis

4.1. Time in Atlantis.

A further examination of Plato's design for Atlantis proves fruitful.

A Greek cubit was one-and-a-half Greek feet and had 24 digit divisions (Dilke, p. 26). A stade therefore contained 400 Greek cubits making the measure of the world equivalent to 86,400,000 Greek cubits: cf. 86,400 seconds in a day.

In the following calculations, "real" pi or $\frac{22}{7}$ can be used. Plato used $\frac{22}{7}$. This is proved in the larger version of this exposition and in *Number and Divinity in Antiquity*. For ease and meaningful outcomes, the fraction $\frac{22}{7}$ is used here.

4.1.1. The concentric ring arrangement of Atlantis was detailed in 2.1. The central island, five stades in diameter, has an area of $19\frac{9}{14}$ ($\frac{275}{14}$) square stades. The area of the largest ring island is $169\frac{10}{14}$ ($\frac{2376}{14}$) square stades.

The quotient of $169\frac{10}{14}$ divided by $19\frac{9}{14}$ is 8.64. A day contains 86,400 seconds and the ancient measure of the world was 86,400,000 Greek cubits. Review the pentagram material in 3.3.1.

4.1.2. The area of the smaller ring island is exactly one-third that of the larger ring island, $56\frac{8}{14}$ ($\frac{792}{14}$) square stades. It is exactly equal to the area of a course for horseracing on the larger ring island. Plato writes:

On the middle of the larger island, in particular, there was a special course for horseracing; its width was a stade and its length that of a complete circuit of the island, which was reserved for it. (Lee, *Timaeus* and *Critias*, pp. 140–1/S117)

Note how the philosopher has drawn attention to this feature and has provided mensural details. This setup is for one of the most unique mathematical feats ever to have come from antiquity. It is dealt with later.

4.1.3. The area of the largest water ring, the outer ring, is $226\frac{8}{28}$ ($\frac{6336}{28}$) square stades. It is equal to the sum of the areas of the two ring islands. This area is, consequently, $\frac{4}{3}$ times the area of the largest ring island. The 4:3 ratio is also found in a 3:4:5-proportion triangle. The significance of this triangle to the nature of Atlantis becomes apparent soon.

4.1.4. The Atlantis ring arrangement, as already mentioned in 2.1. is 27 stades in diameter. The quotient of 216,000 stades, the ancient measure of the world, divided by 27 is 8000, which is 20 cubed.

4.2. Atlantis and the pentagon.

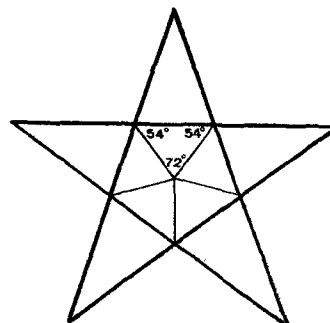


Figure 14: angles in a pentagon.

As can be seen in the above illustration two angles are prominent in the pentagon component: 72° and 54° .

a) In the Atlantis concentric ring arrangement, the area of the largest water ring is $226\frac{8}{28}$ square stades. This is equivalent to the area of a circle that has a radius of $\sqrt{72}$ stades.

b) The area of the largest ring island is $169\frac{10}{14}$ square stades. This is equal to the area of a circle that has a radius of $\sqrt{54}$ stades.

All the components of the Atlantis concentric ring arrangement are dealt with later.

4.3. Dodecahedron.

In the *Timaeus* where Atlantis is first mentioned by Plato, there is a discussion of the five Platonic solids. Plato equates the dodecahedron with its twelve faces, each face a regular pentagon, with “the whole heaven” (Lee, *Timaeus* and *Critias*, p. 78/S55). In *Phaedo*, however, the philosopher tells us, through Socrates, that:

... ‘the earth’s true surface, viewed from above, is supposed to look like one of those balls made of twelve pieces of skin, ...’ (Tredennick and Tarrant, p. 177/S110)

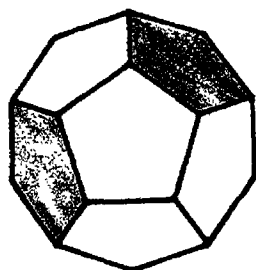


Figure 15: a dodecahedron.

Here is Plato exercising his remarkable imaginative faculties. He chooses the dodecahedron to represent “the earth’s true surface” because information on the size of the planet and the nature of time is bound up in the geometric properties of this unique solid.

4.4. Plain facts.

The other main physical feature of Atlantis, we are told by Plato, is of a great rectangular plain 3000 stades by 2000 stades (Lee, *Timaeus* and *Critias*, p. 141/S118). It was mentioned in 3.5.1. that a stade contained 600 Greek feet. Hence, the plain can be said to measure 1,800,000 by 1,200,000 Greek feet, an area of **2,160,000,000,000** square Greek feet. There are no prizes for guessing the number alludes to the ancient measure of the world. Recall the mention by Herodotus of the cubical block of stone with a volume of 216,000 cubic feet in the shrine of Leto at Buto (3.8.3.) and the area of the lake created by Amenhotep III in 3.8.5.

4.5. Finding time, $\frac{22}{7}$ and much else in 3:4:5 proportions.

The pentagram was not the only geometry involved in Plato’s design for Atlantis. The 3:4:5-proportion triangle had a key role too. (Much more is made of this triangle in *How Plato designed Atlantis and where Vitruvius obtained his model for Man* and in *Number and Divinity in Antiquity*.)

The proportion clearly attracted interest in the ancient world:

- (i) At least eight ancient Egyptian pyramids, including the second largest, the pyramid of Khephren (Khafre) which is next to the Great Pyramid on the Giza plateau, exhibit such proportions. The angle of slope (tangent $\frac{4}{3}$) is about $53^\circ 7' 48''$ (Baines and Malek, pp. 140–1 and Petrie, *TPTG*, p. 202).

(ii) Plato discusses the 3:4:5 proportions in the *Republic* (S546) in relation to the “divine creature’s cycle”; Vitruvius mentions them several times, particularly in the Introduction to Book Nine of *The Ten Books on Architecture*; in *Isis and Osiris (Moralia V)*, Plutarch associates the “three” side with Osiris, the “four” side with Isis and the hypotenuse, the “five” side, with their son, Horus (Babbitt, 135/S373–4).

4.5.1. The geometry in figure 16 below is of great historic interest. In this one diagram, solutions to important ancient mysteries can be found: Atlantis; the pyramids; key metrologies of antiquity. The following is a description of what has taken place in the illustrated geometry. Everything has been realized by the use of straight edge and a pair of compasses — mostly straight edge. It is instructive to study this geometry carefully; there is a kind of rhythm to the process:

- i) ABC is a 3:4:5-proportion triangle. Squares are drawn on the three sides.
- ii) K is the midpoint of BC. K is connected to D on the “four” square by a line. The line crosses AC at R.
- iii) A dotted line continues from K to (lower case) “b” on the base of the “five” square GF. From “b” a dotted line parallel to AC is drawn to (lower case) “h” on BG. The small triangle hGb has 3:4:5 proportions.
- iv) From K a perpendicular line is drawn to L on AC.
- v) From B a line is drawn through L to O on DE.
- vi) A perpendicular line is drawn from A to I on GF. AI crosses BC at M. AM is 2.4 units. MY is constructed to be also 2.4 units. YI is 2.6 units.
- vii) BO crosses AM at N. Through N, a line parallel to AC is drawn from P on AB to Q on BC. PQ crosses DK at U.
- viii) From U a line parallel to BC is drawn to V on AB. VU crosses AM at Æ.
- ix) From M a line is drawn to S on AB; SM is parallel to AC. From S a perpendicular line is drawn to W on BC. SW crosses BO at X.
- x) From K, the middle of BC, a line parallel to AC is drawn to T, which is the midpoint of AB. From T a perpendicular line is drawn to Z on BC.

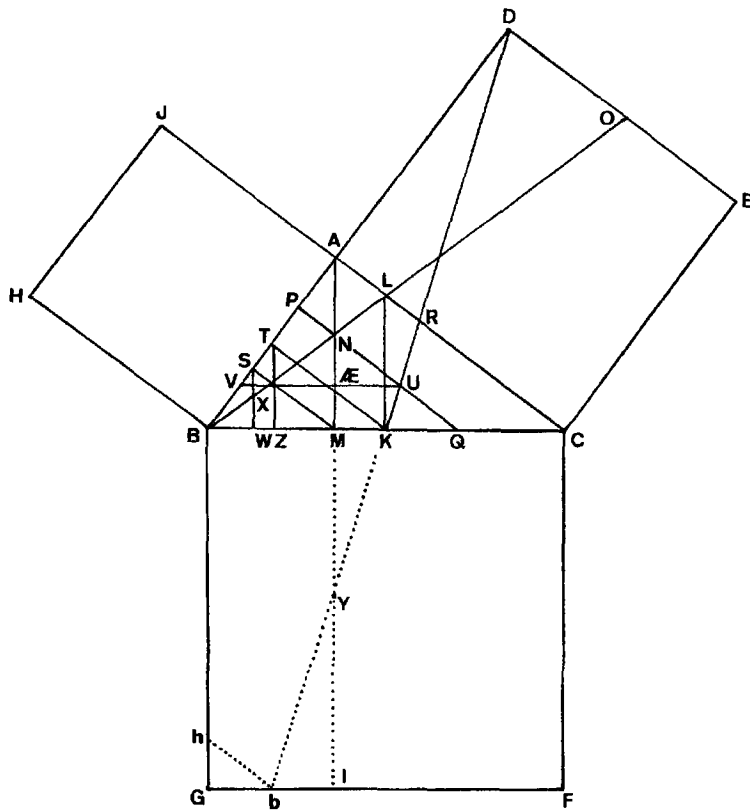


Figure 16.

4.5.2. Features and outcomes

a) The perimeter of ABC measures 12 units ($3 + 4 + 5$): cf. the twelve greater Olympian gods and Plato's many references to the number twelve in his writings: *The Laws*, for instance. A British imperial foot has twelve inches. Most analogue clocks express time in terms of hours numbered from one to twelve and this can be directly linked to ancient Egyptian water and shadow clock designs. A year contains 12 months, just like the ancient Egyptian year.

The perimeter of the entire configuration measures 36 units ($3 + 3 + 3 + 4 + 4 + 4 + 5 + 5 + 5$). A British imperial yard contains 36 inches; a circle contains 360 degrees; an

ancient Egyptian sacred year contained 360 days (Gardiner, p. 203). Also see Adam's remarks in 1.4.1.

The product of 3 multiplied by 4 multiplied by 5 is 60: cf. the sixty-second minute and the sixty-minute hour.

In 3.3.1. it is shown that the sum of the angles in a pentagram is 1440° and that a day contains 1440 minutes; SM measures 1.44 (1.2^2) units. (The pentagon component of the pentagram contains five by 108° ; SB measures 1.08 units.)

b) AM measures 2.4 units and SV 0.24 unit. A day contains 24 hours. The Greek cubit was divided into 24 parts called *daktyloi* (Dilke, p. 26). The Roman cubit and the ancient Egyptian common (short) cubit also had 24 digit divisions. (In *Number and Divinity in Antiquity* it is shown how the Greek cubit was created in the geometry presently under examination and how the geometry, in an astonishing natural process, automatically divides the measure into 24 digit divisions.

c) MK is 0.7 unit. BD is ten times larger and is 7 units. A week contains seven days. QC is 1.4 units: a fortnight contains 14 days. The body of the ancient Egyptian deity Osiris is said, according to Plutarch, to have been cut into 14 parts by his brother Seth and those parts scattered throughout Egypt.

d) Triangle BPQ has three measures: BP 2.16 units, PQ 2.88 units and BQ 3.6 units, a total of 8.64 units. A 24-hour day contains 86,400 seconds. It was shown in 4.1.1. that the area of the larger ring island in Atlantis was 8.64 times the area of the central circular island. In 3.5.1. the entrance to the Great Pyramid was described and it was shown that the original height of the structure, 280 royal cubits, divided by 8.64 accurately produced the height above pavement level of the entrance. SW measures 0.864 unit. All the main traditional numbers associated with the way time is measured have thus been located in full or miniature form: 12; 24; 60; 7; 14; and 360.

e) Triangle ABC has an area of 6 square units. RKC, a scalene triangle, has an area of $1\frac{10}{11}$ square units. The quotient of 6 divided by $1\frac{10}{11}$ is $\frac{22}{7}$, the value often used in mathematical calculations to represent pi, a transcendental number. Its appearance in the design of the Great Pyramid has been much discussed: for example, see Petrie, *TPTG*, p. 183.

KC is 2.5 units and RC is $2\frac{6}{11}$ units. Although a perpendicular line from R to BC has not been drawn to minimize clutter, RK slopes at tangent $\frac{36}{17}$. This leads to a momentous finding.

f) This material is of major historic interest. RC is $2\frac{6}{11}$ units and AR is $1\frac{5}{11}$ units, a total of 4 units. The tangent of right-angled triangle DAR is 2.75. This is readily established. T is the middle of AB 3 units, as K is the middle of BC. AT is, therefore, 1.5 units and DT is 5.5 units (AD 4 + AT 1.5); TK is 2 units; $DT 5.5 \div TK 2 = 2.75$. Triangle DAR clearly has the same proportions as triangle DTK.

RC can be expressed as $\frac{28}{11}$ units and EC as $\frac{44}{11}$ units. Instantly the numbers associated with the Great Pyramid arise: the height of the structure was originally 280 royal cubits and the base was 440 royal cubits square (Petrie, *TPTG*, p. 183).

The first true pyramid constructed was at Meydum, about 40 miles south of the Great Pyramid. It originally had a base **275** royal cubits square and a height of **175** royal cubits (Legon, *Meydum*, pp. 18–9):

$$AD 4 \text{ units} \div AR 1\frac{5}{11} \text{ units} = \mathbf{2.75}. \quad RC 2\frac{6}{11} \text{ units} \div AR 1\frac{5}{11} \text{ units} = \mathbf{1.75}.$$

Two pyramids in one design. It is possible to understand now why Khephren's pyramid, the largest 3:4:5-proportion structure ever built, has been located next to the Great Pyramid. There are other remarkable mathematical reasons as well and they are explicated in *Number and Divinity in Antiquity*.

Something else of interest: AP is 0.84 unit and DP is 4.84 units; 4.84 is 2.2 squared. Since the tangent of right-angled triangle, DPU, like DTK, is 2.75 then PU must be 1.76 units. VU is 2.2 units and PV is 1.32 units. VPU is yet another 3:4:5-proportion triangle. The sum of its sides is 5.28 units. A mile contains 5280 feet or 1760 yards. Furthermore, the perimeter of the base of the Great Pyramid originally measured 1760 royal cubits, its half base 220 royal cubits. TP is 0.66 unit, so is TV: a chain in 66 feet and a furlong is 660 feet. The sum of VU 2.2 units and PU 1.76 units is 3.96 units which is half 7.92 units; a British imperial link measures 7.92 inches and there are 100 links in a 22-yard chain. The yard, as stated earlier, is 36 inches or 3 feet. It is possible to see quite clearly the birthplace of the so-called British imperial system of linear measures in this geometry. Even the number associated with the number of inches in a mile, 63,360, can be found; it is explicated in further developments that are at the far edge of the human imagination in *Number and Divinity in Antiquity*.

The royal cubit is one of the most famous and controversial measures of history. The controversy usually concerns its origin. The matter can now be settled. It derives from the geometry presently under examination.

W. M. F. Petrie thought the best value was one ascertained from the dimensions of the King's Chamber in the Great Pyramid. He estimated the measure, which has **28** digit

divisions, to be 20.620 ± 0.005 inches (Petrie *TPTG*, p. 179). The acceptable range is, therefore, 20.615 to **20.625** inches.

The construction of the small 3:4:5-proportion triangle hGh was described in 4.5.1. (point iii). Gh measures $\frac{165}{224}$ unit. Twenty-eight times $\frac{165}{224}$ produces 20.625. As inches, this is the intended value for the royal cubit. The measure has a clear geometric link to the design for the Great Pyramid as well as the $\frac{22}{7}$ proportion.

The quotient of AB 3 units divided by AR $1\frac{5}{11}$ units is 2.0625.

LK is 1.875 units. If LK were extended to GF, the base of the “five” square, the line would measure 6.875 units; 6.875 is one-third of 20.625. In *Number and Divinity in Antiquity*, the measure as a whole, along with its digit division (not the one shown here), is created in this geometry in a more startling fashion.

g) BP is 2.16 units. The real measure of the world in antiquity was 216,000 stades. The 3:4:5-proportion triangle PAN has sides that measure PN 0.63 unit, AP 0.84 unit, and AN 1.05 units, a total of 2.52 units. Eratosthenes’ fake measure of the world was 252,000 stades. Here we see an analogue of the 216/252 connection in the pentagram described in 3.5.1. WZ measures 0.252 unit.

The ratio 216:252 can be expressed as 6:7. Compare this with the opening chapters of the *Old Testament* in the Bible, *Genesis I* and *II*: God created the world in six days and rested on the seventh.

h) Right-angled triangle ODB has the proportions 7:24:25. OD measures $2\frac{1}{24}$ units, BD 7 units and BO $7\frac{7}{24}$ units ($7\frac{7}{24}$ appears on a calculator display screen as 7.291666...).

BO, as inches, is exactly one-thousandth the measure of the stade (i.e. $7291\frac{2}{3}$ inches), as it was originally understood to be. Two times $7\frac{7}{24}$ inches is $14\frac{7}{12}$ inches (14.58333...). The ancient Egyptian remen has long been estimated to measure around 14.58 inches (Zupko, p. 6 and Petrie, *MW*, p. 5). Recall the appearance of the stade in the Giza pyramid layout described in 3.5.2. The axial distance from the west side of the pyramid of Menkaure to the west side of the pyramid of Khéphren is 7289.5 inches. The rate of accuracy in layout intention is around 99.97 percent.

The ratio between the 20.625-inch royal cubit and the $14\frac{7}{12}$ -inch remen is 99:70, a ratio that closely approximates the irrational number $\sqrt{2}$. (A4-size paper, which measures 297 mm by 210 mm, expresses this ratio.)

i) The stade of $7291 \frac{2}{3}$ inches when divided by 600 yields the Greek/Parthenon foot of $12 \frac{11}{72}$ (12.152777...) inches: cf. the historic estimates of 12.15 inches (Zupko, p. 6 and Petrie, *MW*, p. 5). In *Number and Divinity in Antiquity*, it is shown how the measure of $12 \frac{11}{72}$ inches is generated naturally in the geometry illustrated in figure 16, along with the royal and common cubits, the remen, the Greek cubit, and the Roman foot and cubit. The geometry even generates their digit divisions naturally. The so-called British imperial foot and yard are readily seen: refer to point (a).

j) The ancient measure of the world can now be accurately fixed: 216,000 stades multiplied by $7291 \frac{2}{3}$ inches produces 1,575,000,000 inches or $24,857 \frac{21}{22}$ (24,857.95454...) miles: cf. the modern polar circumference measure of 24,859.82 miles.

k) The Romans divided the stade into 625 (25 squared) Roman feet. Accordingly, the Roman foot measured $11 \frac{2}{3}$ inches (296.333... mm). The measure of the world was therefore equal to 135,000,000 Roman feet. In figure 16, NM measures 1.35 units.

l) BN measures 2.25 (1.5 squared) units: cf. the length of the Parthenon, 225 Greek feet: see 3.5.1.

m) Triangle AMB has sides that measure BM 1.8 units, AM 2.4 units and AB 3 units, a total of 7.2 units. Inside AMB is triangle BMN whose sides measure NM 1.35 units, BM 1.8 units and BN 2.25 units, a total of 5.4 units. Compare this with the material on 72° and 54° in 4.2.

n) The areas of the three water rings, two ring islands and the central circular island of Atlantis are as follows:

Largest water ring $226 \frac{8}{28}$ square stades, which is equal to the area of a circle that has a radius of $\sqrt{72}$ stades; the perimeter measure of triangle AMB is 7.2 units (also see 4.2.).

Larger ring island $169 \frac{10}{14}$ square stades, which is equal to the area of a circle that has a radius of $\sqrt{54}$ stades. The perimeter of triangle BMN measures 5.4 units (also see 4.2.).

Middle water ring: $81 \frac{5}{7}$ square stades, which is equal to the area of a circle that has a radius of $\sqrt{26}$ stades; YI is 2.6 units (MY is 2.4 units, the same as AM; MI is 5 units.).

Smaller ring island: $56 \frac{4}{7}$ square stades, which is equal to the area of a circle that has a radius of $\sqrt{18}$ stades; BM is 1.8 units.

Smallest water ring: $18\frac{6}{7}$ square stades, which is equal to the area of a circle that has a radius $\sqrt{6}$ stades; the perimeter of triangle BTK is 6 units (BT 1.5 + TK 2 + BK 2.5).

Central island: $19\frac{9}{14}$ square stades, which is equal to the area of a circle that has a radius of 2.5 stades: BK is 2.5 units.

o) The ring arrangement of Atlantis is 27 stades in diameter. The circumference of Atlantis is therefore $84\frac{6}{7}$ stades, which is 618,750 inches or 9.765625 miles; 9.765625 is 3.125 squared. In figure 16, LB and LC each measure 3.125 units.

p) In 3.8.1. Herodotus' description of the length of the coastline of ancient Egypt, 3600 stades, was examined. In figure 16 BQ is 3.6 units and the perimeter of the entire configuration measures 36 units.

In 3.8.2. the historian's report on the distance from Heliopolis to Thebes was discussed.

From Heliopolis to Thebes is a nine days' voyage up the Nile, a distance of eight-one *schoeni* or 4860 *stades*. (Marincola, p. 89/H2.9)

9 day's voyage (216 hours): BZ measures 0.9 unit;

81 *schoeni*: BX measures 0.81 unit;

4860 stades: XW measures 0.486 unit;

216,000 stades, the measure of the world: BP measures 2.16, so does AV;

252,000 stades, Eratosthenes' fake measure of the world: WZ measures 0.252 unit and the perimeter of triangle APN measures 2.52 units.

The geometrical and numerical evidence that this is the source of Herodotus' mathematical formulation for the voyage up the Nile is compelling indeed.

q) Eratosthenes' measure of the world, 252,000 stades, was not chosen for use because of numerical and geometric motives alone: there was a didactic reason as well.

In antiquity, the world at the equator turned 216,000 stades every 86,400 seconds (24 hours). Every second, at the equator, the world moved $2\frac{11}{12}$ (2.91666...) stades; $2\frac{11}{12}$ is one-quarter of $11\frac{2}{3}$, the number of inches in a Roman foot; $2\frac{11}{12}$ is one-fifth of $14\frac{7}{12}$ ($\frac{175}{12}$), the number of inches in an ancient Egyptian remen: see point (h) above.

Now $2 \frac{11}{12}$ stades is equal to (multiply by $7291 \frac{2}{3}$ inches) $21,267 \frac{13}{36}$ inches. The latter number is $\frac{1750}{12}$ squared: cf. the number of inches in a remen in the preceding paragraph. Furthermore, $\frac{1750}{12}$ inches is equal to $12 \frac{11}{72}$ BI feet; a Greek/Parthenon foot measured $12 \frac{11}{72}$ inches: see point (i) above.

r) AV measures 2.16 ($6 \times 0.6 \times 0.6$) units. It is the hypotenuse of 3:4:5-proportion triangle AÆV. AÆ measures 1.728 units; 1.728 is 1.2 cubed. VÆ measures 1.296 units (3.6×0.36); the measure of 216,000 stades is equal to 129,600,000 Greek/Parthenon feet. The number 3600 squared (12,960,000) is discussed in Lee's footnote to Plato's reference to 3:4:5 proportions, the "divine creature's cycle" and the number of days in a "Great Year" (Lee, *Republic*, p. 299–300). The perimeter of triangle AÆV measures 5.184 units; 5.184 is 7.2 multiplied by 0.72.

4.6. A racecourse to remember.

The ancient measure of the world was 216,000 stades. It is equivalent to $24,857 \frac{21}{22}$ miles. The latter number can be written as $\frac{70,000,000}{2816}$. If $\frac{22}{7}$ is inverted and dimidiated seven times, the outcome is $\frac{21}{2816}$. All this is worthy of contemplation. Plato has recorded the number $24,857 \frac{21}{22}$ in a memorable way.

In 4.1.2. the following passage from *Critias* was cited:

On the middle of the larger island, in particular, there was a special course for horseracing; its width was a stade and its length that of a complete circuit of the island, which was reserved for it. (Lee, *Timaeus* and *Critias*, pp. 140–1/S117)

Plato has provided an explicit mathematical description of the racecourse. It is one stade wide and it is in the middle of a ring island three stades wide. There is a reason for the formulation: he has created a formidable mathematical conundrum that only advanced students at his Academy or Pythagoreans, or privileged others could understand.

(The calculations here use $\frac{22}{7}$ to represent pi. It is easier to do all this on a calculator that has memory facilities. It is also instructive to do it without the calculator. The $\frac{22}{7}$ proportion is encrypted in the *Timaeus*; the location and method of encryption are explicated in the larger version of this exposition.)

a) The area of the racecourse works out to be $56 \frac{4}{7} (\frac{396}{7})$ square stades which is equal to (multiply by $7291 \frac{2}{3}$ inches twice) 3,007,812,500 square inches.

b) Convert the area to square miles by dividing by 63,360 inches twice (63,360 inches is the number of inches in a mile). The area works out to be $\frac{24,609,375}{32,845,824}$ square mile. On a calculator display screen this will appear as 0.74923908... mile.

c) Divide the measure of the world $24,857 \frac{21}{22}$ miles by $\frac{24,609,375}{32,845,824}$ and the quotient is 33,177.6, which can be expressed as **24 x 24 x 24 x 2.4**.

That Plato or someone else in his Academy conceptualized all this is also worthy of contemplation

4.6. The Pantheon and a Revelation.

Other significant manifestations of the number 216 and thereby the measure of the world can be found in famous architecture and literature

(i) The first example is the Pantheon in Rome, one of the few great buildings of the Roman Empire to survive to modern times basically intact. It is the work of Hadrian, about 126 AD.

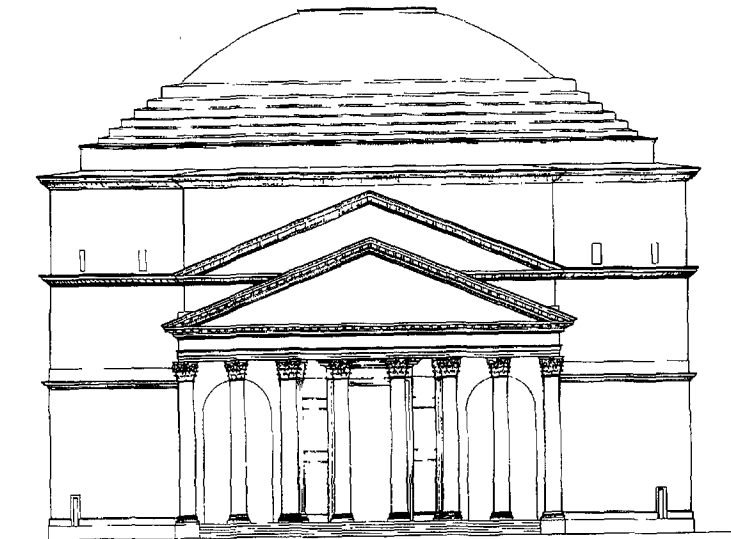


Figure 17: The Pantheon in Rome (illustration courtesy M. W. Jones)

Sir Mortimer Wheeler writes:

It was dedicated to the seven planetary deities and was in effect an architectural simulacrum of the all-containing cosmos. (Wheeler, pp. 104–5)

Amongst the notable architectural features of this structure, normally the first to be seen by the visitor is the portico: see the illustration above. The width of the portico is 108 Roman feet, that is, about 31.99 metres that is about 1260 inches (Davies, Hemsoll, Wilson Jones, pp. 141 and 150) The number 108 is half 216 which is instantly recognizable for its reference to the measure of the world. Also, compare the measure with the angles of 108° in a pentagon as in figure 7.

Furthermore, 108 Roman feet is equal to 86.4 ancient Egyptian remen: the ancient measure of the world was equivalent to 86,400,000 Greek cubits, there are 86,400 seconds in a day. One hundred and eight Roman feet is also equal to 1728 (12 cubed) Roman digits. (A Roman foot had sixteen digit divisions: see Dilke, p. 26.) In figure 16, AÆ measures 1.728 units.

(ii) In the Bible, the last book of the New Testament is titled *The Revelation of St. John the Divine*. It comes down to us in Greek and it is said it may have been composed in Alexandria, in Egypt. The work contains material from a range of sources, some of it ancient Egyptian. Chapter 21, the penultimate chapter, contains numerous mentions of the number twelve. The two most interesting mentions are (1) the visionary New Jerusalem, a giant cube whose length, width and breadth are 12,000 furlongs and (2) a wall 144 cubits high surrounding the city.

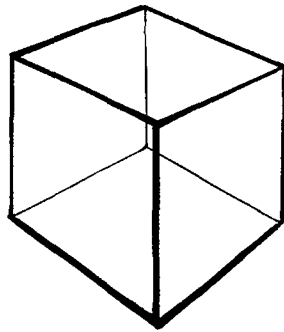


Figure 18: a cube

English translators long ago translated stade as furlong because of a vague similarity in length. It is correct to say that the New Jerusalem cube has sides that measure 12,000 stades. This super solid has a surrounding wall 144 (twelve squared) cubits high. The original text was Greek so it may properly be assumed the cubit is the Greek cubit.

A cube contains 24 right angles and $24 \times 90^\circ = 2160^\circ$. According to the Pythagoreans, the cube consists of “216 lines”: see 3.6.

Each face of the New Jerusalem cube contains 144,000,000 square stades and since the object has six faces, the total surface area of the solid is 864,000,000 square stades. A day contains 86,400 seconds; the ancient measure of the world was equivalent to 86,400,000 Greek cubits. The number 864 has been shown to occur in the pentagram (3.3.1.) and the number 8.64 in the designs for the Great Pyramid (3.5.1.) and Atlantis (4.1.1.).

A cube has twelve edges. An edge of the New Jerusalem cube measures 12,000 stades. The sum of the twelve edges is 144,000 stades, which is equal to 86,400,000 Greek feet.

The wall around the city is 144 Greek cubits high. A cubit (both Greek and Roman) is equivalent to one and a half feet. Accordingly, the wall can be said to be 216 Greek feet high. There can be no doubt as to what the height refers to:

More can be found in antiquity in relation to a square with sides that measure 12,000 stades.

E. A. Wallis Budge compared ancient Hebrew notions of the Underworld with ancient Egyptian ideas of the Duat (or Tuat), the realm of the dead

The commonest of the names, which the Hebrews gave to the abode of the damned, is *GĖ HINNOM*, or *Gehenna*, According to the Rabbis “Gehenna” was created on the second day of creation, with the firmament and the angels, and just as there were an Upper and a Lower Paradise so there were also two Gehennas, one in the heavens and one on the earth. As to the size of Gehenna we read that Egypt was 400 parassangs long and 400 parassangs wide, ...; that Nubia was sixty times as large as Egypt; that the world was sixty times as large as Nubia, and that it would require 500 years to travel across either its length or its breadth; that Gehenna was sixty times as large as the world; and that it would take a man 2,100 years to reach it.

In Gehenna, as in Paradise, there were seven “palaces” ... , and the punishments, which were meted out to their inhabitants varied both in kind and in intensity. In each palace there are 6,000 houses, or chambers, and in each house are 6,000 boxes, and in each box are 6,000 vessels fitted with gall. (Budge, Vol. 1, pp. 273–4)

Gehenna is thus 216,000 times the size of Egypt. The number does not require comment; enough has been said about the measure of the world and $6 \times 6 \times 6$ variants

already.

The mythical area of Egypt, 400 parasangs square, is worthy of note. Herodotus informs us that a parasang (Budge spells it parassang) is equal to 30 stades (Marincola, p. 88/H2.6). Therefore, Egypt was 12,000 stades square, exactly the same size as each face of the New Jerusalem cube. Budge cites Eisenmenger, *Entdecktes Judenthum*, part ii, p. 328 as the source of the information on the size of Egypt.

4.7. Closing remarks.

Atlantis is found. It rises from the depths of myth and speculation to be seen for what it really is: a powerful mnemonic and didactic device for those initiated into certain influential Mystery Schools in antiquity, notably those with Pythagorean and Neoplatonist interests. A large body of data drawn from authoritative ancient and modern sources, from the layout of the Giza pyramids, and from the properties of certain basic geometric shapes known to Plato, Vitruvius and others in antiquity, has revealed a distinctive approach to mathematical design. Unique and inventive mathematical feats previously unknown from the past have been unveiled.

The design source for the Great Pyramid, too, has been located. So has the source for important metrologies of the ancient world, including the so-called British imperial system. Herodotus' marvelous mathematical voyage up the Nile has not only been explicated, it has been placed in the geometry set out in figure 16.

As has been repeatedly demonstrated, the ancient measure of the world was 216,000 stades. It has also been demonstrated that the measure of the world was linked to the nature of time. In addition, it was shown that the measure of the world was established at least as far back as the time of the construction of the pyramids at Giza. No information has come down from the past, though, as to who it was that realized the world was spherical—probably some astute astronomer who witnessed a number of lunar eclipses. There is also no record of the survey method or of the survey work that established the necessary distances for the stade and other measures to be created. The survey work was likely to have been a religious duty as the world was considered a deity, just like the sun or the moon. How many times the task was undertaken until consistent results had been achieved cannot be estimated.

One last surprise remains

5. Golden apples

5.1. Storing knowledge.

From Biblical times, particularly through Christian imagery, the apple has been seen as a symbol of knowledge (cf. the story of Adam and Eve in the garden of Eden). This fruit, a member of the rose family of plants,² has also played an important role in Greek mythology. One myth is especially relevant to the matter of Atlantis and beyond. It concerns Heracles (Hercules) and one of his Twelve Labours.

Using data obtained from ancient sources, Robert Graves, author of *The Greek Myths*, in the opening paragraph of chapter 133 entitled “The Eleventh Labour: The Apples of the Hesperides”, writes (bolded words are the present writer’s emphasis):

Heracles had performed these Ten Labours in the space of eight years and one month; but Eurystheus, discounting the Second and the Fifth, set him two more. The Eleventh Labour was to fetch fruit from the golden apple-tree, **Mother Earth’s** wedding gift to Hera, with which she had been so delighted that she planted it in her own divine garden. This garden lay on the slopes of **Mount Atlas**, where the panting chariot-horses of the Sun complete their journey, and where Atlas’s sheep and cattle, one thousand herds of each, wander over their undisputed pastures. (Graves, Vol. 2, p. 145)

What was so special about this fruit in ancient times? The photograph below says it all. When the apple is sliced crosswise rather than downwards from where the stem connects, the core is revealed as Nature’s pentagram, a geometric shape that encrypts the ancient measure of the world and the way we measure time



Figure 19 Nature's pentagram in the core of an apple.

Thus, the key elements for the philosopher's influential story can be found in an ancient legend that predates Plato's lifetime. Plato has already told us that Atlantis is named after Atlas refer 2.1.1. And we see that the apple-tree is a wedding gift from Mother Earth (Gaea). Who more appropriate? And the apples grow on Mount Atlas a fitting location. Note, too, the mention of horses and recall Plato's course for horse racing (4.1.2 and 4.6). Recall also one of the possible meanings of Apollo's name: apple-man (3.8.4.). The apple truly is a symbol of knowledge.

Notes

1. Zupko incorrectly calculates the Roman foot to be 11.65 inches and the stade as 625 times this value, that is, 606.9 BI feet. Zupko (correctly) rates the 625 Roman-foot stade as equal to 600 Greek feet like other historians of metrology. But he starts out with a slightly incorrect value for the Roman foot 296 mm. The links between ancient Egyptian, Greek and Roman measures are well known and much discussed in works on the history of metrology (see Petrie, *MW*, p. 5, Klein, p. 71 and Zupko, p. 6). The Roman digit was the same as the ancient Egyptian remen digit, around 0.729 inch see the references just mentioned. The Roman foot of 16 digits was therefore around 11.664 inches or about 296.27 mm. Zupko has simply rounded this number to 296 mm and obtained his value of 11.65 inches. Furthermore, when multiplied by 625 it led to his value for the stade measure being out by some seven inches. The well-known ratio (noted by Zupko) of 25/24 between the Greek foot and the Roman foot further

substantiates the corrected measure: the Greek (Parthenon) foot measured around 12.15 inches (Zupko, p. 6 and Petrie, *MW*, p. 5); twenty-four twenty-fifths of 12.15 inches is 11.664 inches.

The so-called Roman foot has been observed in Greece under various guises, including the Nemean “foot”. In a critique of the book *Athletics and Mathematics in Archaic Corinth: the Origins of the Greek Stadion* by David Gilman Romano, British academic David W. J. Gill (see References) writes:

This provides the information that the Nemea *stadion* must have been around 178 m long, judging by the location of the 100-foot marker at a distance of 29.63 m from the starting line; this gives a foot of 0.296 m. (Gill, Internet address in References)

Note how an error in the measure of the Nemean/Roman foot arises: Gill has rounded 29.63 divided by 100 to 0.296. It is more accurate to say the Roman foot was about 0.2963 m (about 11.665 inches).

2. This should interest members of the Rosicrucian Society.

3. Historians of metrology have noted the use of a range of “feet” of varying values in ancient Greece. In all important cases, the present writer is able to demonstrate they are proportionally related to the Parthenon foot, the remen and the ancient Egyptian cubits. The matter is dealt with extensively in “Number and Divinity in Antiquity”, another paper by the present writer. The only Greek foot and cubit discussed in this paper are the so-called Parthenon foot and cubit.”

4. A few remarks on several notable Greek feet measures:

a) The so-called Doric foot of around 326 mm is simply the Parthenon foot of $12 \frac{11}{72}$ ($\frac{875}{72}$) inches multiplied by $\frac{36}{35}$ twice, that is, $\frac{36}{35}$ squared. The Doric foot’s true value is $12 \frac{6}{7}$ ($\frac{90}{7}$) inches (326.57 mm). It has an interesting mathematical function in relation to the measure of the world. As stated in the paper, 216,000 stades is equal to 1,575,000,000 inches (see 4.5.2 point (j)). The quotient of the measure of the world, 1,575,000,000 inches, divided by $12 \frac{6}{7}$ inches is 122,500,000, which can be expressed as $35 \times 3,500,000$.

b) 1,575,000,000 inches divided by the Parthenon foot of $12 \frac{11}{72}$ inches (about 308.6 mm) is 129,600,000, which can be expressed as $36 \times 3,600,000$.

c) The so-called common Greek foot of about 316 mm is more accurately expressed as $12 \frac{4}{9}$ ($112/9$) inches or 316.0888... mm. The quotient of 1,575,000,000 inches divided by $12 \frac{4}{9}$ inches is 126,562,500, which is 11,250 squared.

d) One last example: the Halieis stade of 166.5 metres produces a Greek foot of around 278 mm. The correct value is 10.9375 inches (277 8125... mm) and is $9/10$ of the Parthenon foot or three-quarters of an ancient Egyptian remen. The quotient of 1,575,000,000 inches divided by 10.9375 inches is 144,000,000, which is 12,000 squared.

e) The following website may help with these stade issues:
<http://ccat.sas.upenn.edu/bmcr/1995/95.09.19.html>

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