FOLDED STRUCTURES

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Fields of interest: Kinematically indeterminate structures, discrete geometry (packing and covering problems).

Publications: Tarnai, T. (1996) Symmetry of golf balls, In: Ogawa, T., Miura, K., Masunari, T. and Nagy, D., eds. *Katachi U Symmetry*. Tokyo: Springer-Verlag, 207-214

Tarnai, T. (1996) Geodesic domes· natural and man-made. *International Journal of Space Structures* 11, 13-25.

Fowler, P.W. and Tarnai, T. (1999) Transition from circle packing to covering on a sphere: the odd case of 13 circles. *Proceedings of the Royal Society of London A* 455, 4131-4143.

Tarnai, T and Gáspár, Zs. (2001) Packing of equal regular pentagons on a sphere. *Proceedings of the Royal Society of London A* 457, 1043-1058.

***Abstract**: The paper gives an overview of folding and its applications in some fields of natural and man-made structures, such as symmetric latticed cylinders, braced domes, deployable space structures, insect wings and swollen form of viruses. Application of folding to buckling of thin-walled box columns and expandable polyhedra are also investigated.*

1. ORIGAMI

In traditional Japanese origami, folding of straight line is used, and in this way a great variety of shapes is produced. Origami with much more sophisticated curved lines is also available. Many lampshades are made with this type of folding. Origami occurs also in mathematics, however, the most striking constructions were produced not in Japan but in the US. Kaleidocycles and flexagons were invented in the 1930s. Nowadays a new architectural style is called "folding" with a slight reference to paper folding, though, architectural forms based on origami have been used for a very long time.

2. SYMMETRIC LATTICED CYLINDER

The inextensional buckling form of a thin-walled circular cylinder is the Yoshimura pattern. Probably, it is not an accident that this buckling form was discovered in Japan, because this form of many boxes, lampshades and vases was known before. The Yoshimura pattern, in fact, is a column composed of equal antiprisms. The edge network of the Yoshimura pattern can be considered as a cylindrical bar-and-joint structure If the lower joints are attached to a foundation and the upper joints are free, then for this latticed cylinder, the necessary condition of rigidity is fulfilled. However, it is rigid only if the number of nodes along the circular base is odd. For even numbers, the structure is not rigid (Tarnai, 1980) There are many examples of non-rigid latticed cylinders: braced cooling towers, interstage structures in multi-stage rockets, the timber octagon in Ely cathedral, children climbers, the Gemini telescope, etc.

3. BUCKLING OF THIN-WALLED BOX COLUMNS

Hollow columns with polygonal cross-section usually are made of thin metal plates by folding and welding. Their inextensional local buckling form is like a curved origami. A deeper insight into this buckling pattern is available by studying folding of uniform plane tessellations. Tessellation of Schläfli symbol (3,3,4,3,4) is used as a base of the investigation. The polyhedron obtained by folding leads to a curved origami that approximates the buckling pattern (Tarnai, 1997). Here, neither the polyhedron, nor the curved origami has complete straight-line folds. To maintain zero Gaussian curvature we need some theorems of differential geometry.

4. HIGHER-ORDER MECHANISMS

When folding a plane tessellation into a cylinder the whole structure behaves like a one-degree-of-freedom mechanism with finite displacements. There are bar-and-joint assemblies called higher-order infinitesimal mechanisms, which produce the feeling that they move with finite displacements, but in fact they have only infinitesimal free motions. This feeling is caused by imperfections. In the Madrid codex, there is a sketch of Leonardo about a chain-like bar-and-joint structure composed of T shaped units. The horizontal and vertical segments of the T shape are composed of two parts connected to each other by a joint at an angle. This structure shows an exponential increase in forces in bars. On the other hand, if the member pairs are straightened, then the assembly becomes a higher-order infinitesimal mechanism, which shows an exponential decay of motion.

5. DEPLOYABLE STRUCTURES IN LIVING NATURE

Folded structures are common in nature. At plants, for instance, leaves of trees and petals of flowers are in folded position in buds. When they deploy, usually a growing process is coupled to deployment. At insects, the situation is different. For butterflies, wings deploy at the beginning of their adult period, and their shape is kept until the end of their life. For beetles, however, transparent wings under hard wings are deployed and retracted at every flight. Different species apply different folding patterns.

6. APPLICATIONS IN SPACE RESEARCH

Space structures are usually compactly packaged on earth, and deployed in orbit in space. Packaging of deploying masts, solar panels, membranes and antennas is a basic problem. There exist several different solutions to it. An inextensional folding system of rectangular membranes is the "Miura-ori", where the folded membrane deploys with one degree of freedom. The principle of this folding system came from the analogy with the large displacement buckling pattern of thin elastic plates that was experimented recently by metal-coated shrinking jelly layers. Miura-ori is a real origami that is used also in map folding. If a circular membrane should be folded, then folding about a central hub can be used by rotation.

7. BRACED DOMES

Nowadays many large domes are built for sport arenas. These structures are mainly constructed as space frames with large height. In order to reduce scaffolding, M. Kawaguchi has developed the "Pantadome" construction system, in which the main part of the structure is erected on ground, but some bars are deliberately left out the construction. The assembly in this phase is a mechanism that can be pushed up to the final position, then by adding the missing bars to it the structure becomes complete. This large-scale origami technique is frequently used in Japan and elsewhere. For temporary use, transportable deployable latticed domes (tents) have also been developed. It is a requirement that a stadium or a sport dome is adapted to weather conditions. Therefore, retractable roofs are built. One group of these structures is the "Iris Dome". The latticed domes work as iris of an eye· increasing or decreasing the pupil, that is, the opening at the top of the dome. Hoberman has realized such a structure, but his structure requires very difficult nodes. Recently, F. Kovács suggested a much simpler solution.

8. EXPANDABLE POLYHEDRA

There are several toys in which a polyhedron is transformed to another one by continuous motion. An early construction, the "jitterbug", is due to Fuller. It transforms an octahedron to an icosahedron then a cuboctahedron. Such a structure was constructed in the Heureka exhibition in Zurich about ten years ago. Not long ago, it was experimentally observed that certain icosahedral viruses have a swollen form if pH of the environment changes (Speir *et al.*, 1995). The known expandable polyhedra having icosahedral symmetry, which were developed by engineers and mathematicians (Hoberman's (1991) expanding globe, Verheyen's dipolygonids, Yananose's Juno's spinner), do not provide proper model of the expanding virus. We have started a research to describe the motion of the expandable virus. Until now we could develop a simplified model, an expandable dodecahedron, which however has six additional degrees of freedom those are not required for the investigated motion (Kovács and Tarnai, 2000).

9. CONCLUSIONS

At the beginning, origami itself, and folded configurations were considered only as toys. Later, it turned out that folded structures have serious applications in space research, medical research and architecture. Importance of folded structures will increase in the future.

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References.

Hoberman, C. (1991) Radial expansion/retraction truss structures. United States Patent, Patent Number 5,024,031.

Kovács, F and Tarnai, T (2000) An expandable dodecahedron. In: Gerrits, J.M., ed., Proceedings of the 4th International Colloquium on Structural Morphology. *Delft: University of Technology, 227-234*

Speir, J.A., Munshi, S., Wang, G., Baker, T S and Johnson, J E (1995) Structures of the native and swollen forms of cowpea chlorotic mottle virus determined by X-ray crystallography and cryo-electron microscopy, Structure *3, 63-78.*

Tarnai, T (1980) Simultaneous static and cinematic indeterminacy of space trusses with cyclic symmetry. Int Journal of Solids and Structures *16, 347-359.*

Tarnai, T. (1997) Folding of uniform plane tessellations, In: Miura, K ed., Origami Science and Art Proceedings of the Second International Meeting of Origami Science and Scientific Origami. *Otsu: Seian University of Art and Design, 83-91.*

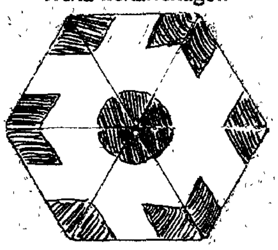
Below is the Power Point presentation of the contribution.

FOLDED STRUCTURES

Tibor Tarnai
Budapest

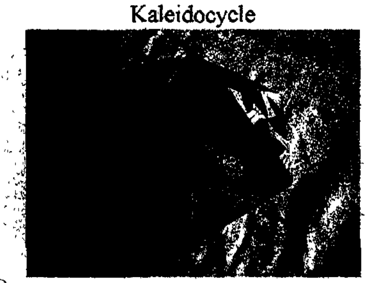
Origami

Hexa-hexaflexagon

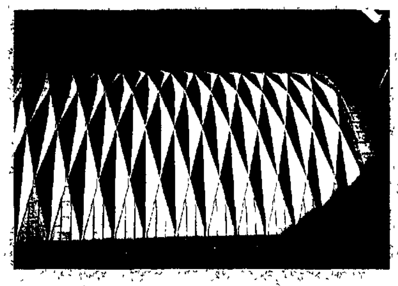


H. Stone, B. Tuckerman, R. Feynman, J. W. Tukey

Kaleidocycle

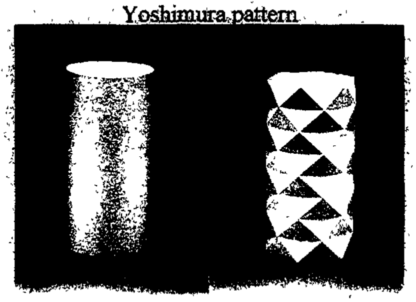


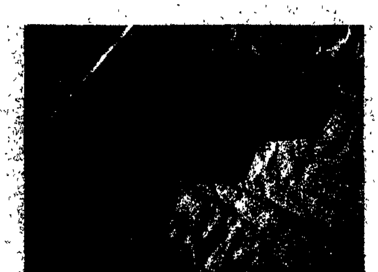
D. Schattschneider, W. Walker

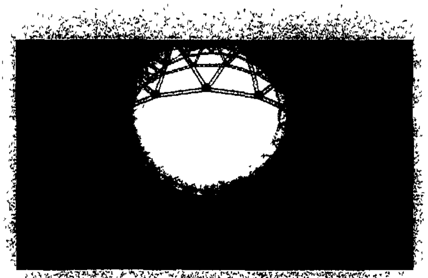
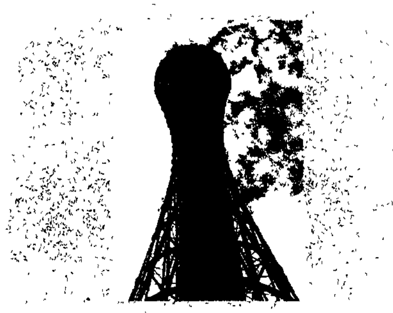


Symmetric latticed cylinder

Yoshimura pattern

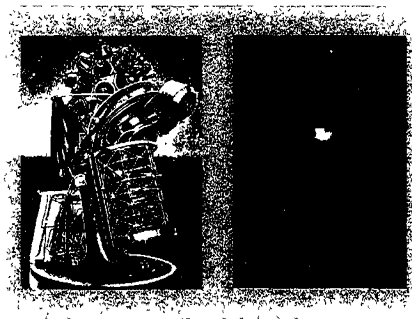




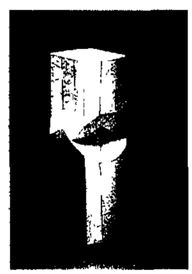



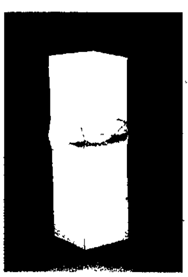






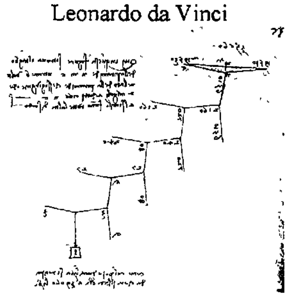
Buckling of thin-walled
box columns

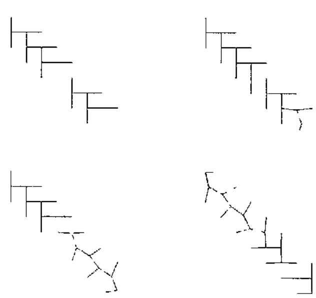


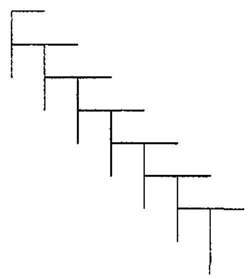


Higher-order mechanisms

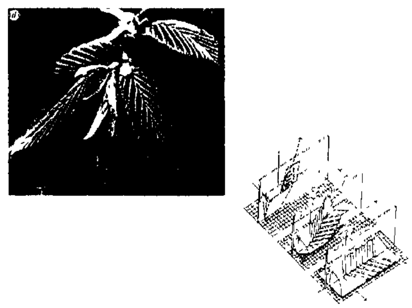
Leonardo da Vinci

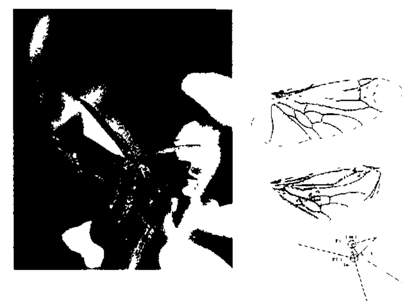


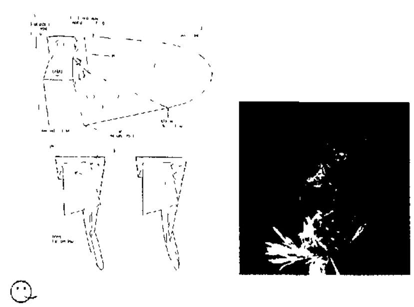




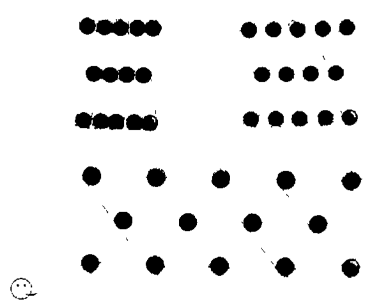
Deployable structures in living nature

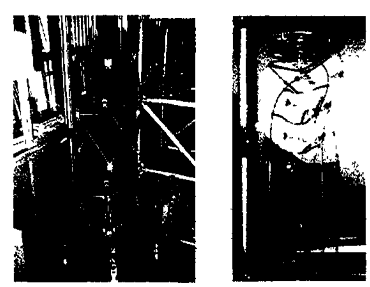


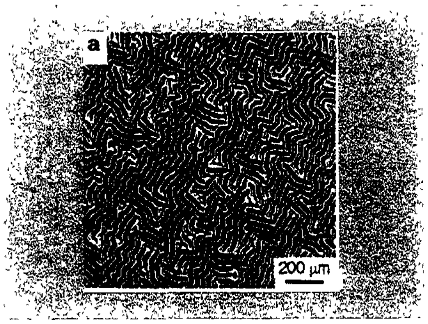




Applications in space research



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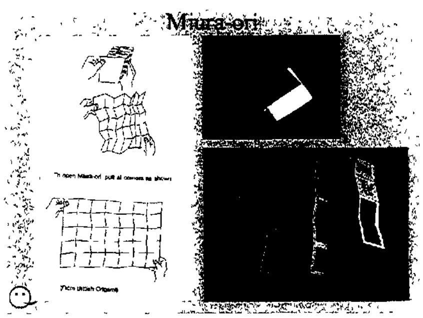
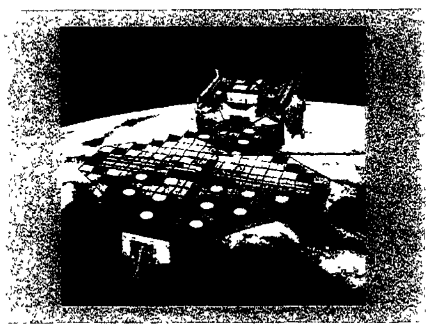
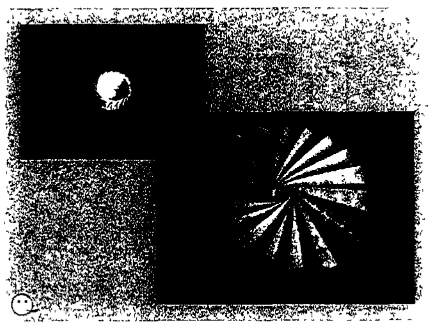
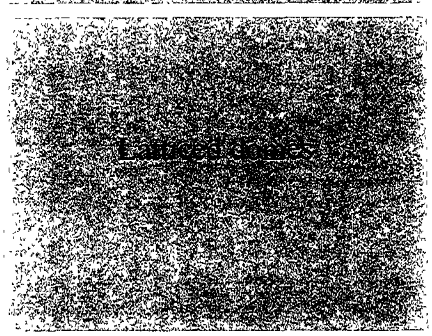
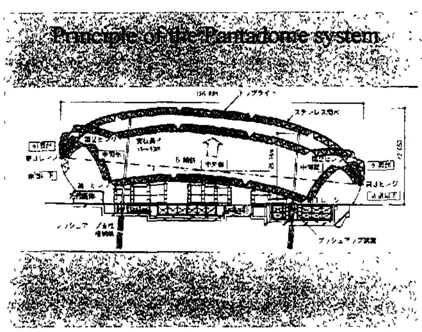
200 μm

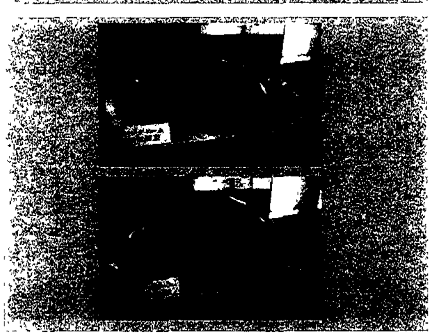

Miura-ori

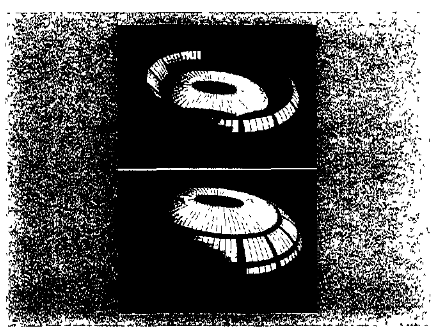


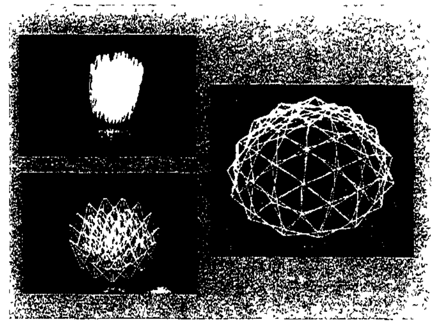


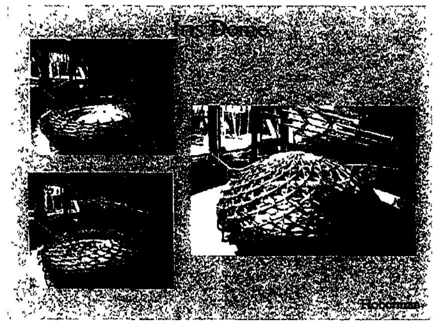

Latticed domes

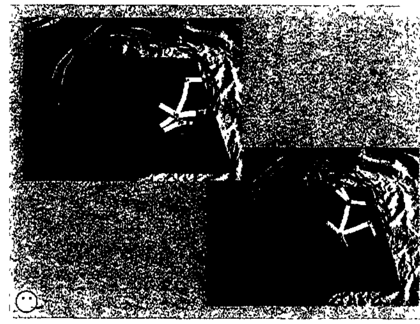

Principle of the Pantadome system

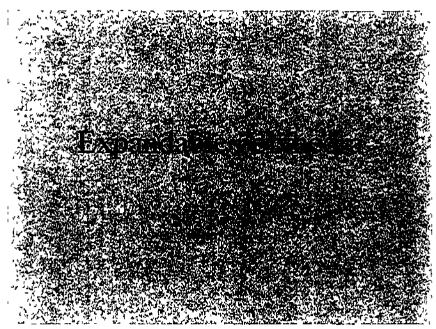


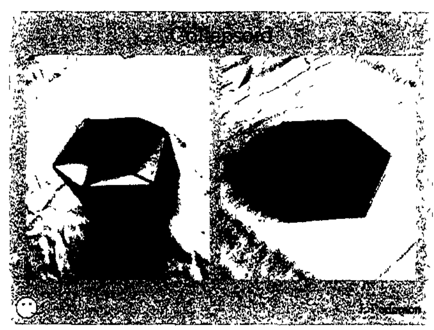


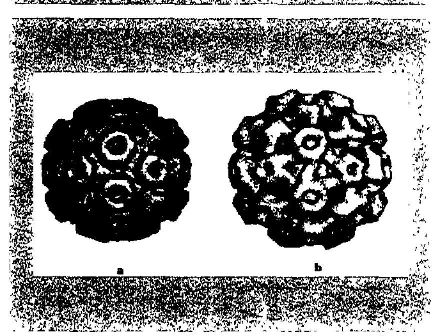


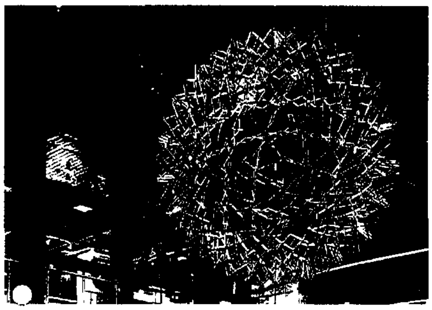

Iris Dome

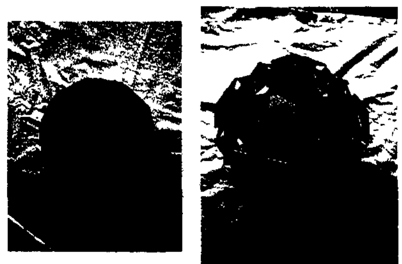


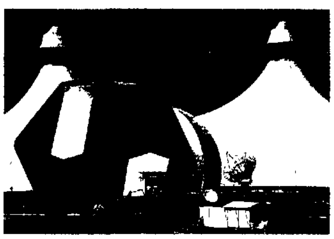


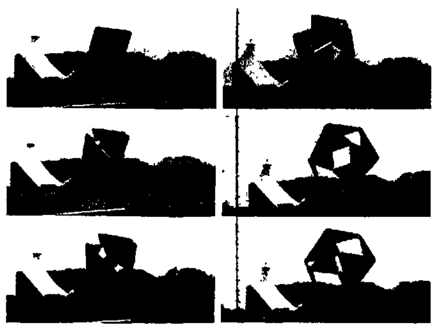


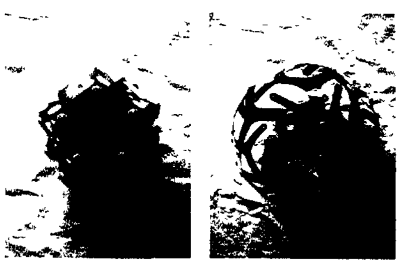




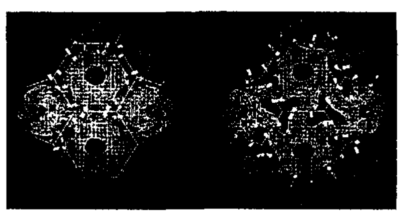


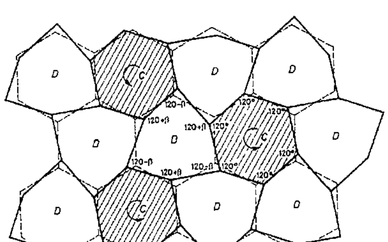




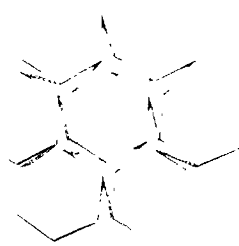


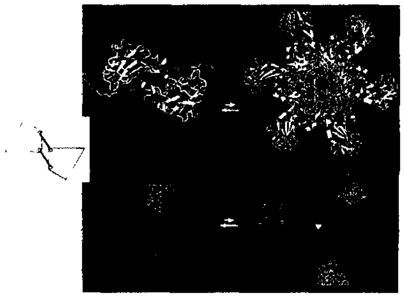
Yananose

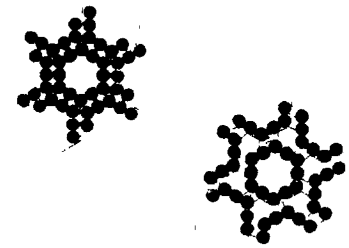


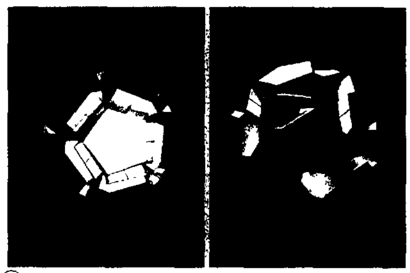


L. Kollár, I. Hegedűs









Conclusions

- It started with toys
- Serious applications in
    - space research
    - medical research
    - architecture
- Increasing significance in the future