THE SYMMETRY OF CRYSTALS

by J.J. Burckhardt

I. Introduction

I received my education at Zürich (Switzerland) and it was also there, where I have been teaching.The athmosphere of this city has formed me. It was also our great poet Gottfried Keller (1819-1890) who has lived there. That's why I beg you to allow me to start my statement by a citation of Keller's: In his autobiographical novel "Der Grüne Heinrich" Keller describes a conversation between himself and a philosopher:

> "Look at this flower", I said to the philosopher, "it is impossible that this symmetry with its counted points and jags, these white and red little lines, this little golden crown in the middle, not has been premeditated! And how delightful it is: a poem, a piece of art, a gay and fragrand joke! All those things could not possibly have made themselves." (Grüner Heinrich, vol. 2, chapter 9, ed. 1916, Cotta, Stuttgart-Berlin, pg. 324.).

These words of the poet show the same conception which lies at the base of my book "Die Symmetrie der Kristalle". I cannot but emphasize that the symmetry which we find in nature is not accidential but premeditated, because such a thing has been made on purpose. It could never have been formed by itself, it is premeditated. At its base lies therefore a mental frame, and we call it "Mathematics".

And now allow me to mention that the mathematitions who have influenced me at Zürich:

1/ I doctorated by Andreas Speiser whose book "Die Theorie der Gruppen von endlicher Ordnung" has influenced my work permanently. Speiser's way of thinking found a big echo at Zürich, I mention his connection with Heinrich Heesch and Wolfgang Gräser for instance.

2/ For a long time I have studied the fundamental work of Paul Niggli (Geometrische Kristallographie des Diskontinuums", which together with the book by Arthur Schoenflies "Krystallsysteme und Krystallstruktur" was the foundation of my own book "Die Bewegungsgruppen der Kristallographie" (1947, ^{2}1966).

3/ Lectures and exercises with Georg Pólya and especially his paper "Über die Analogie der Kristallsymmetrie in der Ebene" were extremely valuable to me.

4/ I was introduced into the forms of the different crystals by Leonhard Weber.

5/ Hermann Weyl showed his connection with our own themes only in later years in his book "Symmetry"; an unmeasurable influence in this direction came from his lectures.

6/ I learned to see the connection between physical processes and their mathematical formulation in the lectures and practica of Erwin Schrödinger.

7/ In the second half of my book I mention the work of my friends who have contributed to the problem of symmetry.

II. The disposition of the book

A. The Origin

1. The origin of the mathematically formulated conception of Symmetry goes back to A.M. Legendre.

2/ R.J. Haüy was the first one who formulated the law of symmetry of the crystals. This means that he not only noticed certain symmeteies - as did many other scholars - but that he noticed that those symmetries were guided by a generally valuable law which he called "Law of Symmetry", in German "Ebenmassgesetz".

3/ From Haüy to Laue. It was Christian Samuel Weiss - who was greatly influenced by Haüy - to whom we owe the nomination of the seven crystal systems. It was only in 1984 that it was discovered that - in 1826 already - Moritz Ludwig Frankenheim had distinguished the 32 crystal classes (J.J.Burckhardt, Die Entdeckung der 32 Kristallklassen durch M.L. Frankenheim im Jahre 1826. Neues Jahrb. Mineralogie Monatshefte 31, 1984. pg. 481 f.). Until 1984 Johann Friedrich Christian Hessel was believed to have discovered them in 1830 as first one. The method which Frankenheim uses is remarkable: He discusses the general linear equation of the surface of a crystal. The treatise was published in the now rare journal "ISIS", edited by Lorenz Oken, the first rector of the University of Zürich, and exponent of natural philosophy. The discoverer of the 32 crystal-classes end his treatise with the remark: "that only a very small part of the 32 classes have been observed". We admire the courage of the writer who published his work nevertheless, believing in the value of the mathematical deduction. Hessel was the first one who drew a graphic picture of the 32 classes. Later on, Gadolin could represent them with the help of the stereographic projection; you can see these pictures in many books. Frankenheim was not only the first who found the 32 crystal-classes, but in 1835 already, he mentions 14 lattices and the 5 networks of the plane. A. Bravais followed him in 1850. Leonhard Sohncke went into a new territory when he showed 14 of the 17 ornaments of the plane, and the

65 space groups which are named after his name. The geometrical part of crystallography is crowned, and at the same time finished, by the work of Fedorov and of Schoenflies. Erhard Scholz gives an overview of the results of these two scholars in a modern way.

B. Transition into modern times

Max von Laue, strongly connected to Zürich, was able to show in marvellous experiments, that the theory of the lattice-structure of crystals is true.

C. The School of Zürich

Paul Niggli saw, that the enumeration of the 230 space groups by Fedorov and Schoenflies did not suffice to interprete Laue's X-Rays diagrams. That's why he wrote his work: "Geometrische Kristallographie des Diskontinuums (1919, [2]1973). Some of Niggli's collaborators worked in this direction: You will find a list of the works of Heesch, Laves and Nowacki in this book. The multi-coloured ornaments are largely treated: At Zürich, the 80 black and white ornaments of the double plane were simultaneously treated with different methods by L. Weber and H. Heesch, and illustrated by Weber. An ingenious method, using windet threads, is shown by Ingeborg Hund-Seynsche. It was Heesch who, as the first one, deducted certain double-coloured space-groups with the aid of geometrical considerations; Burckhardt used for this purpose arithmetical methods. Van der Waerden and Burckhardt gave a methodical instruction of how to findmulticoloured groups.
The author was able to show that the 230 space-groups are not only the result of combinations of geometrical symmetries, but that an algebraic-grouptheoretical structure lies at their base. You can find it if you modify the conception of the crystal-class which Frankenheim and all his successors have been using: The theory of equivalence of quadratic forms uses unimodular transformations with integer coefficients which you will be able to transfer to the crystallography, and which will bring you 13 arithmetical classes in the plane and 73 in the space. Simple laws of group theory enable you then to find to each of them the space-groups. We mention as examples the derivation of the plane groups, illustrated in a table.