FROM ITTEN'S TOWER TO VIRTUAL TOWERS: A GENERATIVE ALGORITHM

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Abstract: *This paper deals with the mathematical reconstruction of different models of Itten's Fire Tower, which appear in his* Tagebücher. *A strong geometrical spirit characterizes the configuration of the towers: consequently, we decided to describe them using affine transformations. Vectors and matrices are the basic elements of linear algebra, which allow us an appropriate and straightforward description of the mathematical interpretation of the towers. Using generative algorithm to build up virtual different towers enables us to compare them and decide which is more practical and more pleasing to the eye. Speaking about the towers planned from other artists around the beginning of the twentieth century, we try to remember some examples, underlining that the spirals are involved as crucial lines.*

1 INTRODUCTION

We were inspired by the numerous towers or monuments planned around the beginning of the twentieth century to create a connection between Mathematics and Arts. With this aim, from the point of view of Geometry, we described and built virtually *The Fire Tower* of Johannes Itten (Marchetti, Rossi Costa, 2002), a monument realized only as a prototype, known by only few photographs dated around 1919-20 (Fig.1a). Carrying out our investigations, we found in Itten's *Tagebücher* drawings and sketches related with other configurations of towers, never realized (Fig.1b, c). We were intrigued by the idea to reconstruct virtually, not only the known prototype, but also other different models, to compare them and discover analogies and differences, in order to establish how far mathematics unconsciously or consciously influenced Itten's aesthetic choices

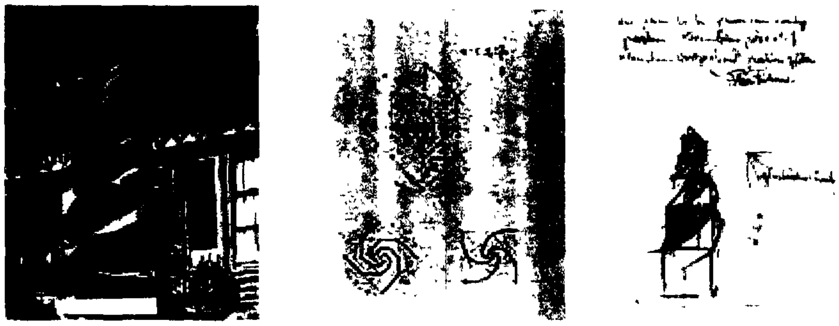


Figure 1: a) Itten's tower prototype, b) and c) Sketches in the *Tagebücher*.

In this paper in Section 2 we recall some information about Itten's production and we briefly describe other typical towers planned at the beginning of the twentieth century. In Section 3 we underline the different configurations of the Fire Tower presenting their mathematical description and a generative algorithm. In Section 4 we mention some points of discussion, which came out during the Mat⊕mium talk, and we give some final conclusions.

2 THE ARTISTIC CONTEXT

2.1 Johannes Itten and the early Bauhaus

The Bauhaus School, founded by Walter Gropius in 1919 in Weimar, involved many important artists of the beginning of the twentieth century, like Feininger, Itten, Kandinsky, Klee, Marks, Moholy-Nagy, Muche, and subsequently students who were particularly clever became teachers, like Albers, Bayer, and Breuer.

The School had as a purpose the mass-production of common-use things but aiming at a fine design, as in the English Arts and Crafts movement. Therefore teachers and students were engaged not only in the artistic field but also in mathematical or geometrical studies, in analyzing forms and colors, in preparing models and respecting proportions. Among the teachers, Johannes Itten as *teacher of form* was one of the most important, bringing in that context his artistic and mathematical formation.

The members of the Bauhaus School were also involved in the philosophical and metaphysical field and made choices characterizing their way of life. In this sense Johannes Itten, very close to Gropius at the beginning of his stay in Weimar, had a big clash with the founder and he left the school in 1923.

During the Weimar period Itten concluded an interesting study on towers, already begun in Vienna before 1919. He had probably different motivations: he planned a bell-tower or a beacon for the Weimar airport as it is evident reading his Diary (*Tagebücher*) (Badura-Triska, 1990). He realized also a prototype now lost, but well known by few photographs.

The promoters of two important Bauhaus Exhibitions in Nuremberg (1971) and in Milan (1995) gave two different reconstructions of the prototype (De Michelis-Kohlmeyer, 1996). In our paper *The Fire Tower* (NNJ- Spring 2002) we examined in detail the photograph of the prototype realized by Itten and were led to accept the Milan reconstruction as being the more faithful to the original idea (Fig.2b). Some of Itten's drawings in the *Tagebücher* (Fig.1c) may well have been used for the Nuremberg model reconstruction (Fig.2a), which is however marked differently from the one presented in Itten's photographs. Other sketches, abandoned by Itten, suggest different forms of the tower, which he never developed beyond this primary stage (Fig.1b).

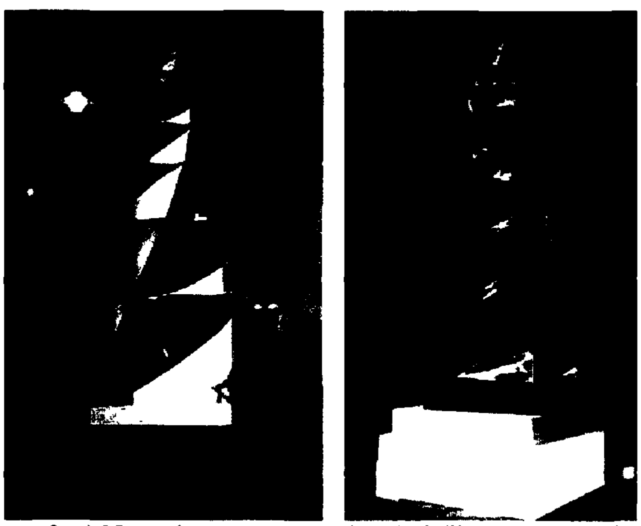


Figure 2: a) Nuremberg reconstruction, b) Milan reconstruction.

We decided to build virtually three different models - Model A: based on Itten's prototype and the Milan reconstruction; Model B: the Nuremberg; Model C: an unrealized plan using the same method, contained in the *Tagebücher*.

2.2 Towers around the twentieth century

As we mentioned before we discover numerous towers planned as monuments by artists around 1900. Some of these contain characteristic cubic or prismatic parts, anticipating the artistic movement called *Cubism;* others present the spiral, an important line full of significance. We think it is interesting to present some examples in order to underline the two different leit-motif, frequently connected in the same monument, as in Itten's towers.
Probably most of these new monuments were inspired by very ancient or less ancient examples, because all the History of Art of any age is rich in buildings having analogous form. We only give examples dated around the turn of the twentieth century.

Figure 3. a) Rodin's tower, b) Obrist's tower, c) Tatlin's monument.

The most famous monument is certainly *The Tour Eiffel* (1889), but we can recall another work of the French artist Rodin, who planned in neoclassical style, at the end of the nineteenth century, the tower named *Tour du travail* (1893-94), in which the spiral appears evident (Fig.3a). It is also important to mention the prototype of Obrist for a monument (1898-1900), a strange cone with oblique axis and spiral motif (Fig.3b).

Quite unusual is Tatlin's *Monument for The Third International* (1919-20): structure conceived in steel bars, forming a spiral developed around an oblique axis, evoking the one of the earth (Fig.3c). The steel bars are connected in a pattern remembering a cage, like the *Tour Eiffel*, but in Tatlin's tower it is not possible to recognize any of the symmetries, which characterize the Paris symbol.

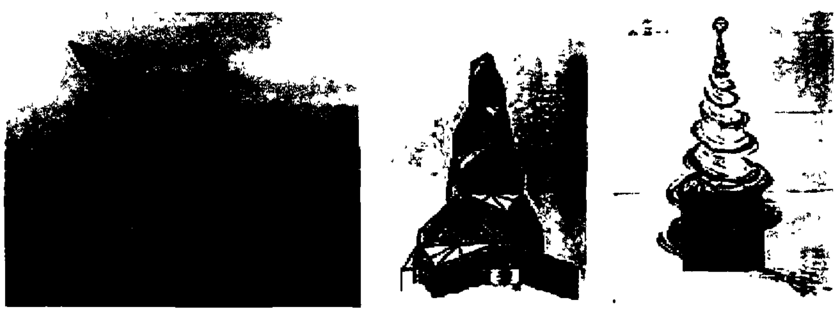


Figure 4: a) Gropius monument, b) Hablik's Exhibition Building, c) Klint's spiral tower.

Gropius and Itten themselves used polyhedra in planning, respectively, the *March Dead Memorial* (1922) (Fig.4a) and the *Composition with dice* (1919) (Marchetti, Rossi Costa, 2002). Among the students of the Bauhaus School we recall two artists involving polyhedra or spirals: Wenzel Hablik for an *Exhibition Building* (1919) (Fig.4b) and Hilma af Klint for a *Spiral Tower* (1920) (Fig.4c)

3 MATHEMATICAL GENERATION OF ITTEN'S TOWERS

Itten studied basic Mathematics and Geometry but certainly he never applied linear algebra techniques. The three projects for the towers presented in the previous section have in common a strong geometrical formation. The virtual prototypes we constructed on the base of different drawings and descriptions follow the same mathematical procedure. Our method was to generate towers by adopting affine transformations of only a few geometric elements. In fact *basic elements* can be seen to appear in the tower many times - always reduced, rotated and translated.

We can synthesize our mathematical process by matrix operators. The points of the structure, intended as points of the three-dimensional space R^3, are represented in a suitable homogeneous Cartesian system *Oxyzu* by four real component vectors $\mathbf{v} = [x, y, z, 1]^T$.

For $-l/2 \le x, y \le l/2, 0 \le z \le l$ the vectors $\mathbf{v}$ describe a cube of side l as the *basic element* of the tower's interior. We apply to vectors $\mathbf{v}$ a $(4,4)$ matrix $\mathbf{M}$, which synthesizes the three fundamental transformations recognizable in the construction of the tower: rotation, scaling and translation.

The matrix $\mathbf{M}$ is the product of the three matrices related respectively to each single transformation.

The product of the matrices $\mathbf{R}, \mathbf{S}$ and $\mathbf{T}$, related to these transformations, gives the $(4,4)$ matrix used in the process:

$$\mathbf{M} = \mathbf{TSR} = \begin{bmatrix} k\cos\vartheta & -k\sin\vartheta & 0 & 0 \\ k\sin\vartheta & k\cos\vartheta & 0 & 0 \\ 0 & 0 & k & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $0 < \vartheta < \pi/2$ is the rotation angle around the $z-$axis, $\sqrt{2}/2 \le k < 1$ is the scaling factor, $l > 0$ the translation factor and

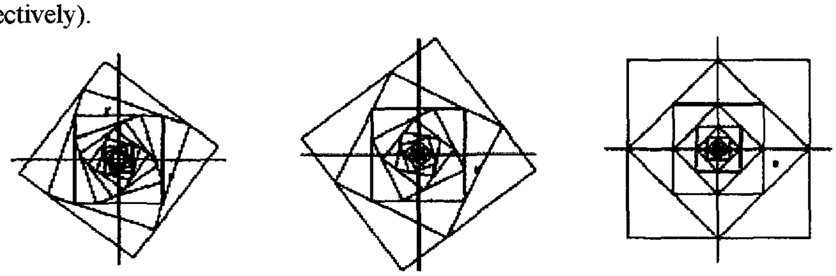
$$\boldsymbol{R} = \begin{bmatrix} \cos\vartheta & -\sin\vartheta & 0 & 0 \\ \sin\vartheta & \cos\vartheta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} k & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

are the corresponding matrices.

The vectors $\mathbf{w} = \mathbf{Mv}$ describe the cube transformed from the *basic element*. By applying this process eleven times, we obtained the twelve cubes of the tower.

A similar procedure can be followed to generate the two supports and the conic surfaces decorating the sides of the tower.

In Fig.5 you can see the projections on the Oxy plan of the cubic system and of the two supports (like in Fig.6 and Fig.7, from left to right, the drawings are referred to the Itten's prototype, the Nuremberg reconstruction and the virtual model with $\vartheta = \pi/4$, respectively).



Model 𝔄 Model 𝔅 Model ℭ

Figure 5: Square projections.

From the detailed mathematical study of the bi- and tri-dimensional figures obtained, we have deduced that the vertices of the squares or cubes belong to arcs of logarithmic spirals, whose equations can be easily evaluated. In Fig.6 the 3D spirals and their corresponding plan projections are shown. The detailed mathematical description, the particular vector-equations and the matrices involved are in (Marchetti, Rossi Costa, Internal report, 2002).

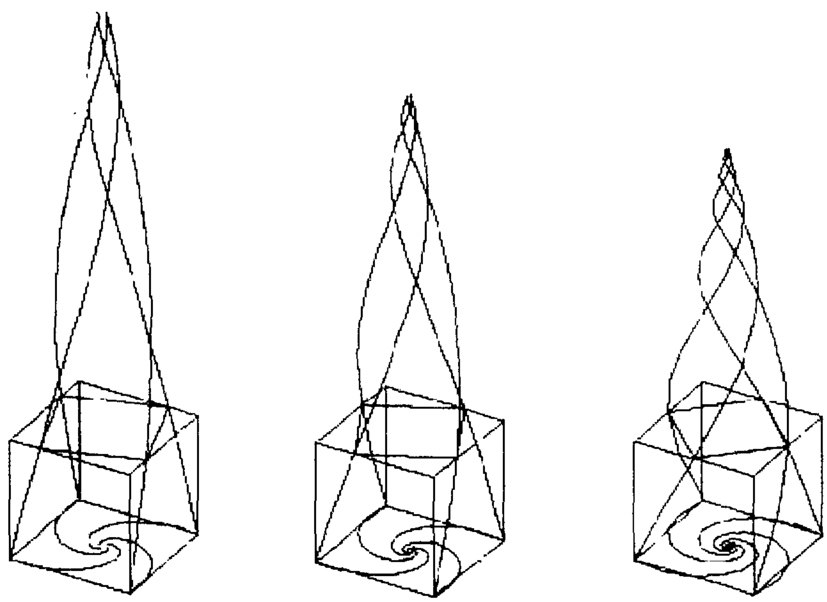


Figure 6: Spirals in 3D.

From the mathematical point of view the different towers are characterized by a different choice of the parameter ϑ, being the value k depending on ϑ. A generative algorithm can give all the infinite models; in all of them the initial idea is recognizable at every step. In this way we have generated (Fig.6) the three towers proposed by Itten, corresponding to the value $\theta = \arctan(1/3)$, $\theta = \arctan(1/2)$, $\theta = \pi/4$, respectively.

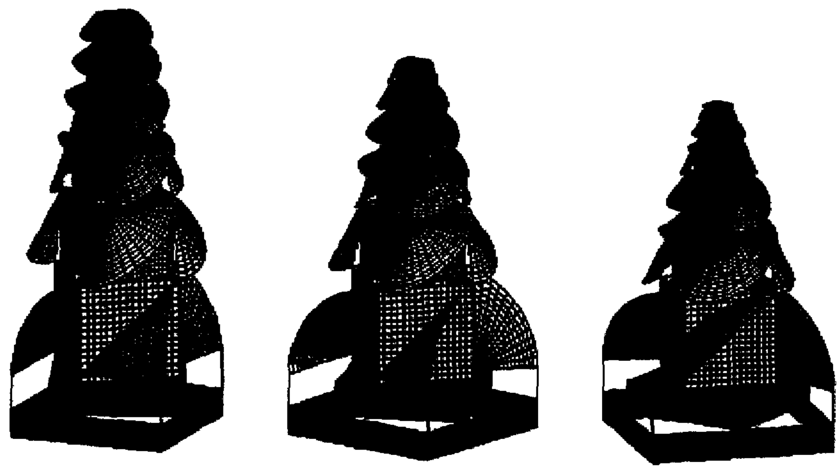


Figure 7: Elements of the virtual reconstructions.

We can observe that the shortest tower is for $\vartheta = \pi/4$, corresponding to $k = \sqrt{2}/2$; increasing ϑ between 0 and $\pi/2$ the height $\widetilde{h}$ of the tower can be represented as a function $\widetilde{h} = \widetilde{h}(\vartheta)$ symmetric with respect to $\vartheta = \pi/4$, having its minimum in $\vartheta = \pi/4$.

In analogy with the ancient question known as *cube duplication*, connected with *Delo's altar*, during the discussion in the *Mat⊕mium* we were asked about the value of ϑ giving at every step a cube of half- volume. We verified that it is possible to solve this problem with two angles ϑ symmetric with respect to $\vartheta = \pi/4$, corresponding to $k^* = 1/\sqrt[3]{2}$. It must be noted that in this case the limitation $\sqrt{2}/2 \leq k < 1$ is satisfied; on the contrary if you would like to have a sequence of cubes with a volume a third of the previous, you can't do it applying this process, as the corresponding k is out of the range.

4 DISCUSSION AND CONCLUSIONS

Our talk gave us the opportunity to examine different aspects of the subject, at the end of the presentation of the work and during informal talks. We were encouraged to give more historical information about the production of artists around the change of the century and about the Bauhaus period.

In the previous section 3 we answer to an interesting question related to the mathematical study of the tower, that it is possible to halve the volume of the initial cube. We appreciate the friendly atmosphere and the easy approach to Mat⊕mium participants.

Going back to the towers, we underline once again that all of them have the same leit-motif. Thinking to the different versions, we built up virtual examples, which tend to be squat (as model **B** and **C**), so we can conclude that Itten's prototype (model **A**) is graceful, soaring towards the sky. Not only is it more pleasing to the eye, but also it is more practical, if as it is thought, it was designed to function as an airport beacon.

The mathematics is always the same; the change of angle gives the aesthetically more pleasing model. The choice of the angle makes the difference!

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