VIBRATIONS OF SYMMETRIC MECHANICAL SYSTEMS

L.Ja.Banach,101830 Moskow, Griboedov Str., 4.
Research Institute of Mechanical Engineering,
USSR Academy of Sciences

Systems,that have geometrical symmetry,find wide application in many branches of mechanical engineering: they comprise various foundations, reduction systems, bladed rotors,etc.

The dynamics of such systems has certain features which make their use "a must".Typical of them, in particular, are: a)independence of various classes of motion (e.g.progressive and torsional);b) existence of a comparatively "quiet" zone - the symmetry centre,which is a node for all torsional vibrations.

Symmetric systems in mechanics have unique features, which strongly influence the work of such systems; they are:

1) technological scatter of parameters, resulting in asymmetry (quasi-symmetric systems);

2) hierarchy of subsystems, each having a symmetry of its own;

3) existence of extended solid bodies with 6 degrees of freedom and indefinite type of symmetry;

4) multidimensional displacements of characteristic points, defining the type of symmetry (generally having 6 degrees of freedom). Certain specific features are introduced due to the employment of finite element method (FEM).

All these features have necessitated generalisation of the existent approaches and creation of symmetry block operators. To this end block projective operators have been introduced, such operators comprising diagonal blocks which allow to account for a multimeasurable nature of system nodes; "equivalent points", chosen for extended solid bodies, are unique in that their displacements are concerted with group symmetry of an entire system.

Block operators of symmetry can be expressed as

$$P_{\mu} = \frac{f_{\mu}}{n} \sum \chi^{\mu} (g^{*})\, g \qquad (1)$$

where n- order of group G; f_{μ} - measure of μ-th representation; χ^{μ} - diagonal matrix, comprised of characters of the μ-th irreducible representation; g - element of group G.

Transformation of coordinates

$$h = P^{\mu}\, y \qquad (2)$$

is equivalent to matrix transformation of the original dynamic matrix of stiffness

$$\overline{D} = (P^{\mu})^{T}\, D\, P^{\mu} \qquad (3)$$

On the power of orthogonality of various irreducible representations, matrix D falls into independent blocks, each of which describes irreducible representation of its own

$$\overline{D} = K - \lambda M \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_n \end{bmatrix} \qquad (4)$$

For quasi-symmetric systems such falling is effected with accuracy down to tiny magnitudes of the order of ε , occurrable in out-diagonal elements of matrix D, which is a sign of weak interrelation between independent classes of motions.

Thus, representing the original matrix as (4), will mean decomposition of system in conformity with independent classes of motions, defined by (2,3); in this sense, (2) can be regarded as generalized forms of virations. Such generalized forms of vibrations are convenient when analyzing multimeasurable systems, since it is unreasonable to follow each form of vibrations separately. This is particularly important for forced vibrations because external forces are usually distributed in conformity with one of subspaces (2), so that only one block remains in (4) which describes this subspace.

To analyze forced vibrations on the basis of the group representation theory it is necessary to decompose external force vector by symmetry operators as

$$F = P^{\mu} F \quad .$$

The analysis of operators (2) permits the following tendencies in symmetric system vibrations be revealed without aid of computers: a) to establish undulating character of vibrations; b) to establish the number of independent classes of motions and their configurations, also the number of multiple frequencies; c) to find out how different classes of motions are interrelated depending on asymmetry distribution; d) to define optimum methods of distributing asymmetric elements under different applications of load; e) to account for symmetry in a hierarchy of subsystems; in this case the resultant operator will be the product of operators for each of the subsystems, i.e. the product of undulating motions for corresponding types of symmetry. In engineering practice such a hierarchy is, in fact, a routine procedure involved in improving design models, in the course of which they become progressively more detailed.

The approach has been used to analyze vibrations in symmetric and quasi-symmetric frames-foundations for power generating plants. Fig.1 shows a damped pentagonal frame; its symmetry group is C .Offered for observation is a finite element model with two intermediate nodes 6-15 on either side.

It is reasonable to choose the coordinate system for each vertex 1-5, due account made of the entire frame symmetry (Fig.1a).

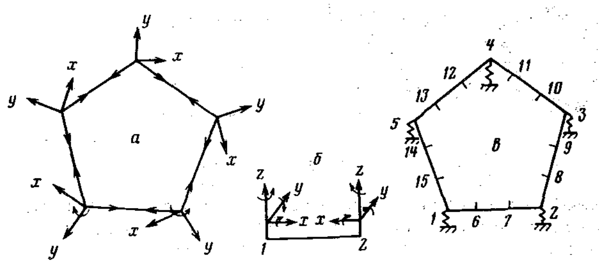

Fig. 1

Then the dynamic matrix of stiffness will assume a simple form, comprising blocks of two types

$$D = \begin{vmatrix} a_{11} & a_{12} & 0 & 0 & a_{21} \\ a_{21} & a_{11} & a_{12} & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{12} & 0 & 0 & a_{21} & a_{11} \end{vmatrix} \qquad \begin{aligned} a_{11} &= \theta_{-\varphi}^{T} k_{11} \theta_{\varphi} + \theta_{-\varphi}^{T} k_{11} \theta_{\varphi} \\ a_{12} &= a_{21}^{T} = \theta_{\varphi}^{T} k_{12} \theta_{-\varphi} \end{aligned} \qquad (5)$$

Character T means transposition, θ_{φ} - matrix of torsion through angle φ . Basis vectors of subspaces form blocks of projective operators P, so that blocks a are transformed similarly to scalars for unimeasurable nodes. The analysis of operators shows that the system has one unimeasurable and two two-measurable representations, i.e. there are 2n/5 two-fold roots,corresponding to representation U_2, 2n/5 two-fold roots,corresponding to U_3, and n/5 - one-fold roots. If additional nodes are present on the pentagon sides, representation (2) will decompose original matrix D as

$$D = \begin{bmatrix} D_{11} & & & & \\ & D_{22} & D_{23} & & \\ & D_{32} & D_{33} & & \\ & & & D_{44} & D_{45} \\ & & & D_{54} & D_{55} \end{bmatrix}$$

when classes of motions for either of subspaces U_2 and U_3 are interrelated. This is attributed to the fact that the forms of vibrations of 1-5 nodes and those of 6-14 and 7-15 nodes are linear combinations of $h_2 \in U_2$ and $h_3 \in U_3$ vectors; generally speaking, they are non-orthogonal to the former, thus explaining the emergence of terms Dij. This conclusion will obviously hold for any finite-element model.

Fig. 2 shows the results of calculation of low natural frequencies and forms of vibrations, whose analysis confirms theoretical conclusions about quantities of multiple frequencies and configuration of vibrational forms. The analysis of amplitudes of vibrations shows that displacement 1-5, 6-8-10-12-14 and 7-9-11-13-15 vary by representations, belonging to one subspace, which means correct choice of projective operators in a block-like form (2).

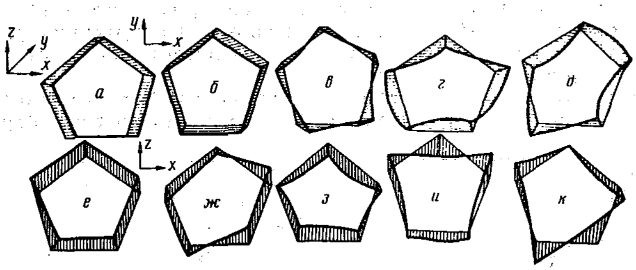

Fig. 2. а,б-21,7;в-24,3;г,д-29,8;е-30,9;ж,з-35,2;и,к-50,8 gz

Study of quasy-symmetric systems. For quasi-symmetric systems the dynamic matrix of stiffness can be presented, after having made group transformations, as

$$\bar{D} = \begin{bmatrix} \bar{D}_{11} & \varepsilon \bar{D}_{12} & \cdots & \varepsilon \bar{D}_{1n} \\ & \bar{D}_{22} & \cdots & \varepsilon \bar{D}_{2n} \\ & & \ddots & \vdots \\ \text{symmetric} & & & \bar{D}_{nn} \end{bmatrix}$$

which reflects weak interrelation between independent subspaces. A solution for natural and forced vibrations can be represented as converging series by exponents ε .

At forced vibrations the interrelation of various subspaces is defined both as distribution of external force and as occurrence of resonance states at $\omega = \omega_{ext}$ in some of a subsystem.

Fig.3 shows a damped square-shaped frame intended for power shaped frame intended for power generating equipment. As a rule it is practically impossible to make the frame perfectly symmetric: there is always a scatter of parameters leading to interrelation of vibrations and occurrence of beatings as a result of detuning of multiple frequencies. Let us consider the interrelation of vibrations caused by the scattering of dampers' stiffness parameters.

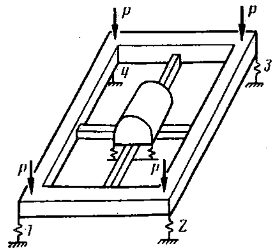

Fig. 3

The frame stiffness matrix after being expanded by symmetry operators

$$\begin{bmatrix} 4a_{11}+8a_{12}+\Sigma H_i & H_1-H_2-H_3-H_4 & H_1+H_2-H_3-H_4 & H_1-H_2-H_3+H_4 \\ & 4a_{11}+8a_{12}+\Sigma H_i & H_1-H_2+H_3-H_4 & H_1+H_2-H_3-H_4 \\ & & 4a_{11}+\Sigma H_i & H_1-H_2+H_3-H_4 \\ \text{symmetric} & & & 4a_{11}+\Sigma H_i \end{bmatrix} \qquad (6)$$

where a_{11} and a_{12} are determined similarly (5).

When distributing asymmetry to type U_2, i.e. $H_1=-H_2=-H_3=H_4$ as evident from (6), interrelations arise between subspaces U_1 and U_2.

Hence:

- if the external force is distributed to type U_2, i.e. $F_1 = F_2 = -F_3 = -F_4$, then resonance states may arise on natural frequencies in subspaces U_1 and U_2; in this case there will be no torsional vibrations about axes x and y.

Similar conclusions can be drawn for other cases of asymmetry and external force distribution.

As obvious from the example cited above, frame asymmetry appreciable influences the dynamics of system as a whole: progressive and torsional vibrations are generally not decoupled; beatings arise as a result of detuning of natural multiple frequencies; placing the rotor into geometrical centre of symmetry does not help in dividing shapes of vibrations. However, knowing the scaterring of dampers characteristics and distribution of external force, one can, employing the approach offered, arrange the dampers in such a manner that their asymmetry will not induce any interrelation between certain classes of motions, ensuring good vibration resistance of the object. These qualitative conclusions can be obtained without computer-assisted design, by merely analyzing the projective operators.

REFERENCES

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