MULTIDIMENSIONAL SYMMETRY AND ITS ADEQUATE GRAPHIC-ANALYTICAL REPRESENTATION IN THE SYSTEM "MAN-MACHINE-ENVIRONMENT"

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Every "man-mashine-environment" system has many degrees of freedom, is multidimensional and as a rule symmetrical one (Frolov, 1979). It is actual therefore to try to describe multidimensional adequate graphic-analytical information, its symmetry, "isomery" (this term means geometrical ambiquity and is taken from chemistry) etc. Such adequate representations are well known in 1- and 2-dimensional cases. So 1-dimensional representation

$$Y=f(X)=X_1=i_1 X_1=e^{i_1 \alpha_1}$$

corresponds to a point on the real axis (a circle means the importance of the Nature i.e. of a "mathematical dimensim" of a number, for example: a number is reel, imaginary or other one). The birth of 2-dimensional adequate representation may be set at 1673 when John Wallis suggested the geometric representation of complex numbers by points in a plane:

$$Y=f(X)=X_1+X_2=i_1 X_1+i_2 X_2=1\cdot X_1+\sqrt{-1}\cdot X_2=e^{i_1\alpha_1+i_2\alpha_2}$$

"Mathematician expected that the extension from the complex number with $n=2$ to $n=3$, would be childs play, but considerable time elapsed before they found that no such extension seemed to be possible without violating a rule of ordinary algebra" (Moon, 1986).

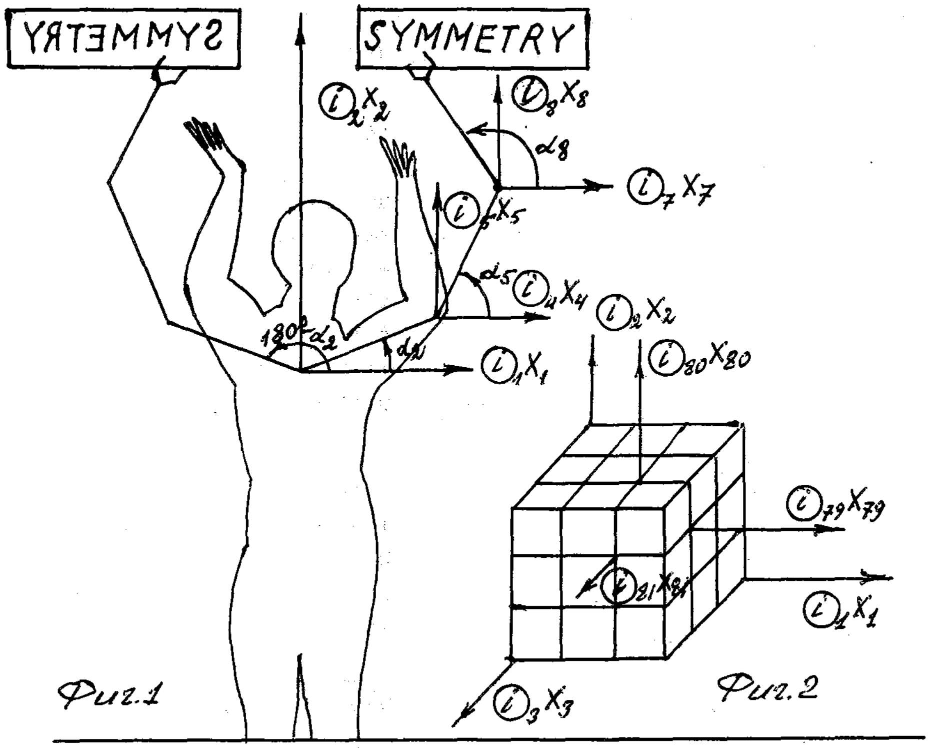
With the aim of simplicity we do not describe here the curvlinear coordinates and symmetries (Petukhov, 1981; Bunin, 1971, 1985) and describe only multidimensional Descartes coordinates (Bunin, 1971).

What is the cause for beauty and power of complex numbers? This cause is for different Nature of axes. The result numbers after algebraic operations may be again distributed on corresponding axes: reel part of result on reel axes, and imaginary one-on imaginary axes. But such a distribution is impossible if the Nature of numbers is the same. The base of n-dimensional representation here is the same. We also use unit numbers of different Nature

$$i_1=1\ ,\quad i_2=\sqrt{-1}\ ,\quad i_3=\sqrt{-1}\cdot{}^{\sqrt{-1}}\!\diagup{-1}\qquad \text{etc.,}$$

where (Bunin, 1967, 1985) $i^{\overset{2}{1},4,7,10\ldots}=+1$; $i^{\overset{2}{2},3,5,6,8,9\ldots}=-1$.

The symbol $\diagup$ denotes here "superroot" (Bunin, 1967) inverse to "superpower" $\diagup$ (for instance $_2 2^2={}^3\!\diagup_2=16;\quad {}^3\!\diagup 16=2$).



Let us consider a simple exsmple of multidimensional symmetry: symmetrical 9-dimensional arms of a robot Fig.1 (partly taked from the well known Weil's book "Symmetry"). n =9 independent values of coordinates corresponds to one point on Fig.1, for example to grab holding the word "SYMMETRY", as usually take place in multidimensional coordinates. Analitical exponential representation analogous to that of 1- and 2-dimensional one is

$$y = f(x) = e^{i_1\alpha_1 + i_2\alpha_2 + \ldots + i_9\alpha_9} + e^{i_1\alpha_1 + i_2(180^\circ-\alpha_2) + i_3\alpha_3 + i_4\alpha_4 + i_5(180^\circ-\alpha_5) + i_6\alpha_6 + i_7\alpha_7 + i_8(180^\circ-\alpha_8) + i_9\alpha_9}$$

We put on the Fig.1 $\alpha_1 = \alpha_4 = \alpha_7$ (all lines are equall); $\alpha_2=30^\circ$, $\alpha_5=60^\circ$, $\alpha_8=120^\circ$; and $\alpha_{3,6,9}=0$ was taken to demonstrate how may be <u>convoluted</u> n-dimensional picture on the flat screen of display (we hawen't multidimensional screens now). Analogous method of representation was described in application to another objects: 3-dimensional spiral (Bunin, 1985), athom (Bunin, 1971) etc. It is interesting to applicate this method to analyse of a Rubic Cube rotations, which obviousely needs in 81-dimensional coordinates (Fig.2).

This quantity of coordinates may be decreased because of multidimensional symmetry of Cube, its parts and movings. It must be noted that our results are absolutely in no contradiction to so-called "Fundamental theorem of algebra", which ban going out "field of complex numbers" only by using of power polinomial, e.g. the operation of 3-d step (power, root, logarithm), and said nothing about possibility or impossibility of such going out by using more powerful operations, for instance, 4-th step ("superpower","superroot","superlogarithm"),etc. Let us consider an example of <u>convolution</u>. The result of an experiment have often the form of a system of n equation with unknowns $X_1, X_2, \dots, X_n$. If we write this system by units of dimensions $\ell_1, \dots, \ell_n$, we become:

$$q_1 = a_{11}\ell_1 X_1 \dots a_{1n}\ell_n X_n$$

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$q_n = a_{n1}\ell_1 X_1 \dots a_{nn}\ell_n X_n$, solution of which is

$$\ell_1 X_1 = \frac{\Delta \ell_1 X_1}{\Delta} \qquad \dots \qquad \ell_n X_n = \frac{\Delta \ell_n X_n}{\Delta}$$

Convolution of this solution on 3-dimensional screen comprise a set of three coordinates $\ell_1 X_1, \ell_2 X_2, \ell_3 X_3$ which definits the origin of other three coordinates $\ell_4 X_4, \ell_5 X_5, \ell_6 X_6$ etc. More dence convolution take place if we use 2-dimensional screen and a set of pairs of such coordinates. The graphical result on the screen of display may show klasters, symmetry, decomposition and help in operate in systems "man-machine-environment".

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