

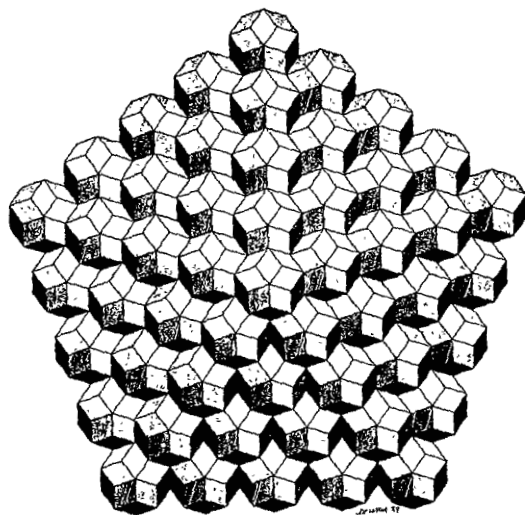
For

Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

II.



Edited by Gy. Darvas and D. Nagy

*Buda
pest*

August 13-19, 1989

hungary



Duality, Nonlinearity, Asymmetry and Symmetry
in Lattice Dynamics

Morikazu Toda

Professor emeritus, Tokyo University of Education

Home Address: 5-29-8-108 Yoyogi, Shibuya-ku, Tokyo, Japan

In my study of lattice dynamics, I have been helped by the ideas such as duality, nonlinearity, asymmetry and symmetry. I would like to present how I met these ideas, how they led me to some discoveries, and to see that many theories can be related by these ideas. I was first led to (1) the concept of duality which relates different one-dimensional lattices with the same frequencies of normal modes. In turn, it led me to (2) the discovery of integrable nonlinear lattice with asymmetric interaction potential. Recently I am interested in (3) a lattice with asymmetric potential and asymmetric kinetic energy (symmetric with respect to kinetic and potential energy terms), which describes asymmetric time sequence, such as stochastic process, chemical reaction, and some environment problem.

1) One-dimensional dual lattices

One of the problems I was interested in lattice dynamics was the effect of impurities on the normal mode frequencies (spectrum) of a linear lattice. We sometimes met different lattices with the same spectra. In this connection, I found the following duality theorem [Toda 1965]:

If we replace masses by springs, and springs by masses, we can obtain another lattice with the same spectrum as that of the original lattice, if certain relations are satisfied between the masses and the force constants of the springs.

These lattices can be symbolically as

A)K U K U

B)U*K*U*K*.....

with kinetic energy terms K, K^* , and potential terms U, U^* :

$$\begin{aligned} K_j &= \frac{p_j^2}{2m_j} & U_j^* &= \frac{k_j}{2} (s_j - s_{j+1})^2 \\ U_j &= \frac{k_j}{2} (x_j - x_{j-1})^2 & K_j^* &= \frac{1}{2m_j^*} r_j^2 \end{aligned}$$

We note that the kinetic energy is symmetric with respect to the momentum p_j or r_j , and the potential energy is symmetric with respect to the relative displacement $x_j - x_{j-1}$ or $s_j - s_{j+1}$. The condition of duality (equivalence) of two lattices A and B is

$$\dots m_1 k_1^* = m_1^* k_1 = m_2 k_2^* = m_2^* k_2 = \dots$$

The simplest equivalence of dual lattices can be achieved by the replacement

$$\frac{1}{m_j} = k_j^*, \quad \frac{1}{k_j} = m_j^*,$$

$$p_j = s_j - s_{j+1}, \quad x_j - x_{j-1} = r_j.$$

2) Nonlinear lattice

The idea of the dual lattice was extended to nonlinear lattices [Toda 1966]. For simplicity we consider a uniform lattice with the kinetic energy term $K_j = p_j^2/2m$, and the potential energy term

$$\hat{U}_j = \phi(x_j - x_{j-1}).$$

the corresponding dual lattice has the potential energy term $U_j^* = (s_j - s_{j+1})^2/2m$, and the kinetic energy term

$$\hat{K}_j^* = \phi(r_j)$$

$\wedge$ means nonlinear. $\hat{K}$ is a kind of kinetic energy with a mass which depends on the momentum r_j of the lattice B. We see that the lattices A and B symbolically expressed as

$$\begin{aligned} \text{A)} \quad & \dots K \hat{U} K \hat{U} K \dots \\ \text{B)} \quad & \dots U^* \hat{K}^* U^* \hat{K}^* U^* \dots \end{aligned}$$

are equivalent (behave the same).

Because the lattice B seemed simpler than the lattice A, I worked on B, and was able to find an integrable lattice [Toda 1967].

This lattice has the asymmetric potential

$$\phi(x) = e^{-p-1+p}$$

It was found that this nonlinear lattice is perfectly integrable [Lunsford and Ford 1972, Hénon 1974, Flaschka 1974].

3) Lattice asymmetric in time and space

Recently I am interested in a lattice with asymmetric kinetic energy as well as asymmetric potential energy. This lattice can be expressed symbolically as

$$\dots \hat{K} \hat{U} \hat{K} \hat{U} \hat{K} \dots$$

The dual lattice is the same to the original lattice (self-dual). Motion in this lattice is asymmetric in time and space. That is, the motion to the right is different from the motion to the left. This strange behavior comes from the asymmetry of the kinetic energy term.

Further, it turned out that this lattice is equivalent to a system considered by Kac and Moerbeke [Kac and Moerbeke 1975] as a model for a certain stochastic process, and to a special case of the Lotka-Volterra model [Lotka 1956, Volterra 1931] for conflicting species, or environment problems.

References

- 1) M. Toda 1965: J. Phys. Soc. Japan 20 2095.
 - 2) M. Toda 1966: Prog. Theor. Phys. Suppl. 36 113.
 - 3) M. Toda 1967: J. Phys. Soc. Japan 22 431, 23 501.
 - 4) M. Toda 1969: J. Phys. Soc. Japan, Suppl. 26 235.
 - 5) G.H. Lunsford and J. Ford 1972: Phys. Rev. 188 73.
 - 6) M. Hénon 1974: Phys. Rev. B9 1921.
 - 7) H. Flaschka 1974: Phys. Rev. B9 1924.
 - 8) M. Toda 1988: RIMS Kyoto Univ. in press.
 - 9) M. Kac and P. van Moerbeke 1975: Adv. in Math. 16 160, Proc. Nat. Acad. Sci. U.S.A. 72 1627.
 - 10) A.J. Lotka 1956: Elements of Mathematical Biology, Dover, New York.
 - 11) V. Volterra 1931: Leçons sur la theorie mathematique de lutte pour la vie, Gauthier-Villars, Paris.
- General reference; M. Toda :Theory of Nonlinear Lattices 2nd ed. (Springer-Verlag 1988).