

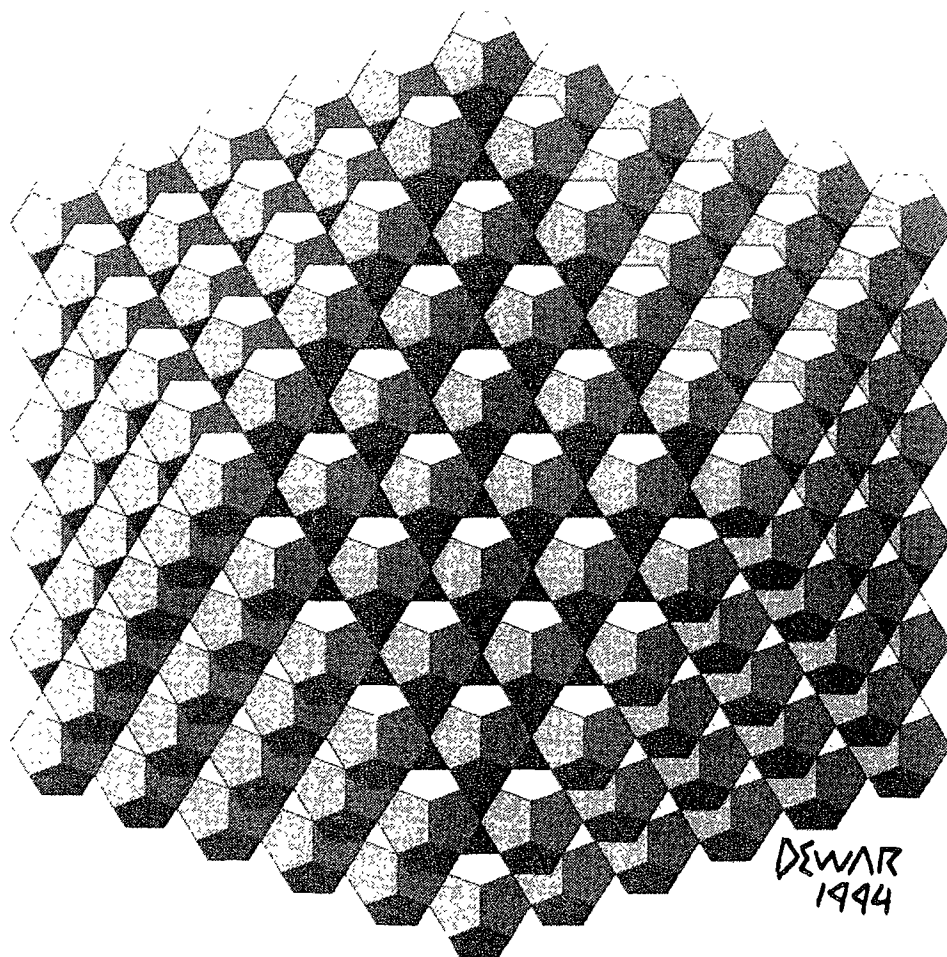
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FUNDAMENTAL SYMMETRIES IN MODERN PHYSICS AND THEIR LIMITS

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The concept of symmetry is familiar from geometry where it denotes that certain geometrical properties "look the same" under some transformations. Symmetries also exist in physics where they similarly mean that a physical entity (position, time, theory, etc.) remains invariant when some of its parameters are recombined. A notable example, in this respect, is the special theory of relativity. Thus, if space and time coordinates are replaced by another combination of these coordinates the theory remains invariant.

Such invariances bring symmetry to play a dominant role in physics because they allow it to single out of the complexities of nature certain structures which are characterised by regularity and repetitivity. Indeed, whenever a physical entity is invariant under symmetry operations there usually exists a corresponding conservation law. The space-time symmetries of nature are perfect examples here. Thus, homogeneity and isotropy of space and homogeneity of time are responsible for the conservation of linear momentum, angular momentum and energy, respectively. These are *continuous* symmetries. *Discrete* symmetries also exist. Such are space-reversal, **P** (parity), and time-reversal, **T**. **P** refers to reversing the spatial coordinates (x, y, z) of a physical entity into $(-x, -y, -z)$ and **P** symmetry means that nature does not distinguish between these two coordinate systems; **T** refers to the indifference of nature to the direction of time, i.e., past and future are indistinguishable.

The time-reversal symmetry is of special interest since it seems to be violated in our daily life. Indeed, whereas the equations of classical physics, relativity and quantum mechanics are symmetric (invariant) under time-reversal, there seems to be a fundamental time-asymmetry, a "Time Arrow", in many large scale phenomena in nature. This is particularly noticeable in the second law of thermodynamics. According to this law, entropy, which measures the manifest disorder of a system exhibits a direction, namely, it increases with time. This apparent violation of the underlying time-reversal symmetry, however, can be reasonably explained by the statistical nature of thermodynamics.

The above examples are in the realm of classical physics. However, symmetries are especially important in the "microworld", particularly in elementary particle physics. Indeed, here symmetry arguments together with the mathematics of symmetry, namely, group theory, provide one of the most powerful means in modern physics for developing physical theories. This strength originates perhaps more in what symmetry forbids than what it allows. This is especially noticeable in quantum numbers and selection rules. Some of these quantum numbers are easy to grasp: the mass, the electric charge or the spin. Others have no classical analog. Such are, for example, charge conjugation, C , and isotopic spin. C refers to viewing particles and their antiparticles as indistinguishable except for their charge and isotopic spin means that the proton and the neutron are viewed as indistinguishable particles (nucleons) differing only in charge. P in quantum mechanics may also be added to these examples. Indeed, here P means space-reversal of coordinates of wave functions with the result that such a function may assume two states: either even (+) P or odd (-) P , and this has no classical analog.

With respect to the symmetries of P and C puzzling symmetry violations were discovered: of the four known interactions (forces) in nature they are symmetries of only the strong interaction (the interaction that holds the nucleons together), the electromagnetic and the gravitational interactions but not of the weak interaction (the interaction that is responsible for radioactive decay). In addition, a certain phenomenon was found in which T is also violated. This occurs in the decay of K^0 -mesons.

These discoveries are of great significance since it was an "unquestioned" assumption that these symmetries are always exact and fundamental. However, as these discoveries show, they are only approximately so. Another concern related to symmetries is the idea of the existence of exact fundamental symmetries which are subsequently "spontaneously broken". This was a crucial idea in unifying the weak and electromagnetic interactions into one theory, the electroweak theory, in which these two interactions are viewed as two manifestations of a single electroweak interaction. The theory is based on exact symmetries ("gauge symmetries") whereas the lack of symmetry which is generally encountered at the observed level, is attributed to the phenomenon of "spontaneous symmetry breaking" (this idea is also behind the attempts to unify all the interactions of nature into one scheme: the "grand unified theory" (GUT)).

This idea is related to the consideration of the state of the temperature of the universe. Thus, when the ambient temperature of the universe was greater than a certain critical temperature, the physical phenomena respected full symmetry but below this temperature symmetry breaking occurred spontaneously and left only partial symmetry. This is analogous to the phenomenon of ferromagnetism. Thus, if a bar of iron is above a critical temperature the atoms are at random motion and have full rotational symmetry but when the temperature drops below this critical value the atoms spontaneously align themselves collectively into a certain direction and the state then has only axial symmetry. Thus, the full rotational symmetry is spontaneously broken.

The above idea explicitly assumes the existence of *exact fundamental* symmetries which are only *subsequently* broken. So, why should one be concerned about these symmetries? The problem is that it raises the following essential question. Is the symmetry indeed to be taken at the fundamental level and the breaking of this symmetry as something more superficial, or is the symmetry itself superficial, being an apparent and approximate phenomenon emerging out of underlying ingredients that do not actually possess this symmetry (Penrose 1993)? As a matter of fact, other routes exist in which effectively the currently accepted electroweak theory may be achieved but where the symmetry is considered as being superficial and not of fundamental nature (Llewellyn Smith 1973).

Thus, the previously mentioned violations of **P**, **C**, **T** and the idea of symmetries which are spontaneously broken raise, respectively, the following concerns about fundamental symmetries in physics: 1) that such symmetries may be exact only within a limited domain, and 2) that underlying symmetries of a theory may not be as fundamental as they are assumed to be even if such assumptions are behind a theory which is fruitful. Therefore, any discussion of the outstanding success of fundamental symmetries in modern physics must necessarily also address these concerns. The present paper will proceed along these lines.

Llewellyn Smith, C. H. (1973), in *Links between Weak and Electromagnetic Interactions*, Rutherford Lab. Report RL-73-018.
 Penrose, R. (October 1993), *Nature's Biggest Secret*, The New York Review of Books, p. 80.

MATHEMATICAL MODELS OF SYMMETRY IN MUSIC

POZZI ESCOT

Jacob Bronowski writes that discoveries of science and the works of art are both explorations and explosions of a hidden likeness, that we see only the surface beauty of nature and life. The Pythagoreans conclude that proportional number-patterns rule the essence of all things. These patterns are the basis of dynamic symmetry and the intuitive realization of them in the creative mind results in those explorations and explosions. Being responsive to the mathematics which connects and balances this symmetry means developing the critical mind and the aesthetic experience. Music has an internal logic or system of proportions governing its phenomena; music's events are of number and progressive divisibility affects the composition down to the smallest aspects and partitions of the overall structure. Again Bronowski writes that Leonardo Da Vinci was occupied with the logic of the processes he saw in people and machines; and he looked for the hidden structure because it expressed that logic and symmetry. This presentation discusses musics from Gregorian Chant to the modern Gyorgy Ligeti analyzing the specific symmetrical formations of their respective designs. Slides are used showing original mathematical models determined by precise statistical information of the score.

The following extended abstract takes three examples of music. We begin with an Antiphon by 12th century composer Hildegard von Bingen. The medieval Gothic world had inherited the legacy of St. Augustine's thought. For him music and architecture were congruent forces since both were the result of a vision of number -- an attempt to repeat the harmonious consonances of an unattainable world. Geometry meant aesthetic perfection and was considered the link between God and earthly realm. It was Abbot Guy Charlieu (a pupil of the Abbot of Clairvaux, St. Bernard) who in his Regulae de Arte Musica writes that music must sound the great Gregorian virtues of symmetry and balance, namely the virtues of mathematics. The analysis of Antiphon "Sed diabolus" brings out its commensuration.

There are clearly delineated six phrases divided by:

- onset/decay on the Final, priority pitch D, of the Dorian mode (phrases 1,2,5)
- onset/decay on the Final and Dominant pitch A (phrases 3,4)
- onset on the fifth D-A dyad (phrases 1,3)
- text semantics and syntax
- "Sed diabolus
in invidia sua
istud irrisit
qua nullum opus dei
intactum
dimisit"

These six phrases of 9, 16, 9, 12, 6, and 16 attack-points each result on remarkable proportions:

- 6,9,12 have arithmetical mean $([a+c])/2=b$
- 6,9,16 have harmonic mean $(2ac/[a+c]=b)$
- the special Golden Mean, negative and positive, occur at salient points of the Antiphon's musical flow -- negative (.382x68 total number of attack-points), divides the Antiphon into two and four phrases; the positive (.618x68), at the second new B pitch in the fourth phrase after 2/3 of the phrase's 12 attack-points and just after the apex of the Antiphon has been sounded
- identical extended decay (E-D-C-D) for phrases 1,2,3, and 6

The fourth phrase, whose text mentions the work of God, presents a triangular graphic notation in its deployment literally suspended from the Final D⁴ and does not end on the Final like all the other five phrases, but on the Dominant, repeating this A on the word God and sounding it on register four, five times. No other phrase sounds the B pitch and the A is only heard in phrases 2 and 3 in register three and the one A⁴ is in the opening dyad of the Antiphon. Other significant symmetries are:

- the highest Final D5 is at exactly 26 equal attacks (negative Golden mean) distance from the very first lowest point and Dominant pitch A3
- the D-A dyad opening the first and fourth phrases divides the Antiphon into two exact halves of 34 attacks each (9+16+9 and 12+6+16)
- the presence of the lowest pitch in the first half balances the presence of the highest pitch in the second half
- two phrases in each half are pentatonic (1,2,4,6) and phrases 3, 5 are each hexatonic and tetratonic
- the extended onset/decay of both halves are the same, only a slight register shift marks the second half extended onset (first half is D-A-D-C/E-D-C-D in register four; second half, the beginning D-A is followed by D-C shifted to register five, the decay is the same).

The first Prelude for piano by Frederick Chopin is divided into four phrases of 8 equal measures each plus 2 added to the last phrase to prolong the Prelude's final cadence.

Both the first and third phrases have cadential endings, the second however, runs into the third one to stress and enrich the Prelude's climax occurring here. The Prelude is a spiral of harmonious growth -- a gnomonic growth where an evident development remains homophetic; namely, a sonic design similar to itself. Through its 34 equal measures the Prelude presents an exquisite dynamic symmetry entirely dependent upon the Fibonacci additive series. Crucial musical events take place at measure 8 where the only lowest attack is sounded; at measure 13 where the first important and permanent change of the harmony is sounded and the precise moment where the chromatic ascent from register one begins; at measure 21 where the only highest and loudest climatic

event take place; at measure 34 concluding the Prelude. The Golden Mean (positive) occurs at the climatic point; the negative at the first change of harmony. Both lowest and highest poles are related spatially/temporally with a .618 ratio. It is at the particular moment where the ascension to the climax is finally given its thrust onward that the French pianist Alfred Cortot (who edited Chopin's Preludes) indicates a *stretto* (a sudden overlapping thematically). This coincides with precisely the division of two equal temporal parts of the Prelude (16+16) if the two extra measures of repeated arpeggiated Tonic (priority pitch) cadence at the end are not considered, since they are unnecessary to the Prelude in terms of language and gestural dimensions. However, these two measures are imperative to balance the temporal dynamic symmetry of additive Fibonacci numbers (8+13+21+34).

Abraham Moles, the French psychologist, refers to music perception as a musical message broadcast in a succession of packages of originality of varying size. With our last example, in this extended abstract, we touch the modern world. Opus 11, No. 1 of Anton von Webern for piano and cello is a chain of eight differentiated packages, all lasting barely one minute, held together by perfect ratios of dynamic symmetry and completely separated from each other by silence, thus obtaining a non-linear surface. There is an exact bi-partite division where every single detail of the sonic frame in the first half presents its mirror equivalence in the second although the music is different in each half:

- a total of 33 pitches each with
- 27 equal durations of which 18.5 are sounded, 8.5 silent
- four chords
- four packages of originality of varying size (6+9+6+6, 3+6+6+12
- eight different pitches for the cello
- four packages having ranges adding up to 139 (100 cents from one adjacent frequency to the other or number of chromatic steps), 38+47+22+32, 35+29+47+28
- the same number of pitches for registers 2,5,6 while 3 and 4 balance each other
- onsets and decays on silence
- the same tritone-sounding interval for its four packages
- balancing at equal distances the lowest and highest pitches of cello and piano, F#2-Eb6 and Eb2-F#6

The varying size packages are in multiples of three and the Fibonacci additive series from 1 to 55 guide the sonic motion through prominent musical events:

- at 1 of equal durational values there is silence
- at 2 there is again silence
- at 3 the lowest cello pitch is sounded
- at 5 there is again silence
- at 8 the highest pitch of the piano is sounded

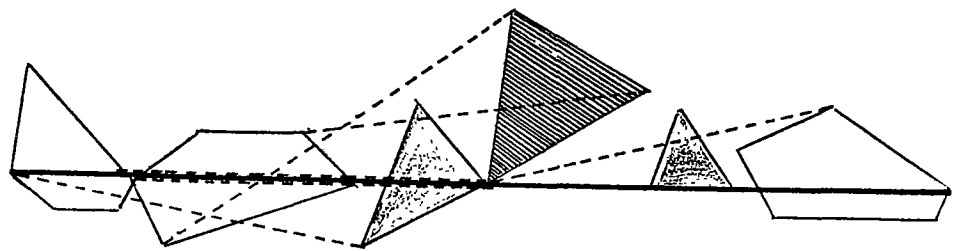
- at 13 the first slowing down of the musical flow appears
- at 21 the climax takes place
- at 34 the last slowing down and the sounding of the lowest piano pitch
- at 55 the work ends (although the entire Opus 11, No. 1 lasts for 54 equal durational values, there is however, 3 slowing down passages for 9 equal values and one speeding up for 6; the extra 3 slowing values adding one value to the actual 54)

Other symmetries are present in this extremely short work:
the coefficient 8 summing up vital constructive features seven times:

- 8 packages or phrases
- 8 chords
- 8 different total time durations
- 8 velocity changes
- 8 different dynamics
- 8 different pitches for the cello in each of the two parts
- 8 as the frequency interval separating the boundaries of the smallest package from the work's own boundaries!

The negative Golden mean happens at the most relevant event of the work, in package 4 where the climax is heard as a loud, noisy, highly dramatic accident that continues the speeding up of the previous package before abruptly changing to slowing down. The positive takes place in package 6 where the musical gestures are heard as if distributed in space and time; where the two most significant pitches of the work, the interchangeable low/high Eb-F# are heard (the lowest Eb for both the piano and the work for the first time) in succession initiating an upward motion of almost two octaves within the limited time duration of the package itself.

Opus 11, No. 1, with less than a minute's span of time, creates a multiplicity of correspondences which outright gear the composition to perfect commensuration, and thus a game of dynamic symmetry fascinating to hear.



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complex plane of "Sed diabolus", abscissa = temporal flow
of equal attack-points, coordinate = frequency
geometrical notation of the six phrases show vertices of onset/
apex/nadir/decay
broken lines = Golden Mean equivalences among vertices (negative)
solid black line = the Final priority pitch D4